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An Article on Nonlinear Waves In Plasmas

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Abstract: *A Plasma is a partially or fully ionized gas, where ionization and recombination are balanced to keep the ionized gas to be charge-neutral as a whole. If the charged neutrality is broken, a large electricfieldmaybelocallycreatedtodriveplasmaunstable.Inthissense,charge neutrality (quasi-charge neutrality) is an essential ingredient of plasmas. Plasma, being a collection of charged particles, respond strongly and collectively to electromagnetic fields through long-range Coulomb interaction.*
Keywords*: Solitary Waves, Ion Acoustic Waves, Arbitrary amplitude, Pseudo potential approach*

I. Introduction

Plasma: A Plasma is a partially or fully ionized gas, where ionization and recombination are bal-anced to keep the ionized gas to be charge-neutral as a whole. If the charged neutrality is broken, a large electric field may be locally created drive plasma unstable. In this sense, charge neutrality (quasi-charge neutrality) is an essential ingredient of plasmas. Plasma, being a collection of charged particles, responds strongly and collectively to electromagnetic fields through long-range Coulomb interaction. Plasma can be broadly

Characterized by the following basic parameters:

- The density of the neutral particles ,nn.
- The densitie of the electro nandions,ne,i.

In the quasi-neutral state of plasma, the densities of the electrons and of the ions areusually equal, ni∼ ne= n and n is usually called the plasma density. The energy distri-butionsoftheparticles,fn,e,i

Foranyionizedgastobetermedasplasma,following conditions must be fulfilled

L>>λD
ND>>1 (1)
ωτ>1

ne^2

where $\lambda_D = \left(\frac{\varepsilon_0 KT_e}{ne^2}\right)^{1/2}$ is the characteristic length over which any small electrostatic perturbationmaybeneutralised,whichiswhy,itiscalledtheDebyescreeninglength.TheDebyescreeninglengthmustbemuchlargerthanthedimensionoftheplasmas

TotalnumberofplasmaparticlesinaDebyesphereisgivenby

$$N_D = \frac{4}{3} n\pi\lambda_D^3$$

and

$\underset{\tau}{\omega}_\tau = \left(\frac{ne^2}{m_e\varepsilon_0}\right)^{1/2}$ isthecharacteristicfrequencyofstandingoscillationsoftheelectrons.

Waves in Plasmas: A plasma contains a wide variety of waves because of its fluid like behavior and also because of its long range interaction between the particles in it.It is well known thatplasmaisadispersivemedia.Againfromthestudyofplasmaoscillationitisobviousthat plasma waves can propagate in a dispersive media. So in plasma medium plasma particles and waves can coexist and they can interact with each other and the oscillationcan occur. The plasma waves have a direct application to human information exchangebymeansofradiowaves.Wavesarealsoimportantforlarge-scaleprocessesinnature.
Thestudyofplasmawavesinspaceplasmasinvolvesthemeasurementofthecharacter-istic frequencies of the plasma in order to understand basic properties of the plasma suchasitsdensityandtheeffectofthemagneticfieldwhichmaybethreadingtheplasma.Sincethe charged particles in a plasma respond to static and oscillatory electromagnetic fields,strong interactions can occur between these plasma waves and the underlying chargedparticles in the plasma. These strong interactions are often called instabilities.

Electronplasma oscillations at the plasma frequency (sometimes called Langmuir waves) are oneexample of an instability in a plasma. In many cases, plasma waves and instabilities areimportantinunderstandingthestateoftheplasma,theevolutionofenergyandthefluxofplasmainamagnetizedplasma,and anumberofotherinterestingphenomena.

II. Application of Plasma Physics

Becauseplasmasareconductiveandrespondtoelectricandmagneticfields and can be efficient sources of radiation, they are usableinnumerousapplicationswheresuchcontrolis needed or when special sources of energy or radiation are required. Plasmas appearon small scale in our natural environment in lightnings, fire, auroras etc. From altitudesabove, say, 100km the fraction of charged particles is steadily increasing and the neutralcomponent is negligible above 400km altitude. The gas in the upper parts of the iono-sphere is completely ionized.On cosmological scales plasma is entirely dominating theuniverse,withsafeestimatesgivingthatatleast99%ofallmatterisintheplasmastate.Itsstudyisthusofbasicimportancefor ourunderstandingofnature.

Industrialapplicationsofplasma: Plasmas are also important for many technologically significant processes. It is importantfor welding, plasma etching, in high voltage circuit breakers, light sources etc. The inter-action between charged particles and matter is a question of central importance for thesemi-conductor industry, and plasma effects have entered this field also. In many chem-ical processes it is advantageous to rise the temperature in order to get higher reactionrates.Theresultcanbeapartialionizationofthegasandsocalled“plasmachemistry”is becoming an important field, opening possibilities for production of new materials. Ad-vanced methods for waste disposal rely on processing materials in the plasma state. Aninteresting recent development concerns the study of dust-plasmas, i.e. charged dust (orother macroscopic particles) in a plasma environment. This problem is relevant to astro-physics as well as technological processes. Interplanetary plasmas have imbedded a non-trivial amount of macroscopic dust particles, where a significant fraction is charged. Thedynamic properties of the surrounding plasma is modified significantly by these chargedparticles.

Plasmafusion: Fusion research is one of the most important reason for interest in plasma physics.Thepeaceful use of fusion energy is expected to require a control of light elements at tem-peraturesinexcessofmillionsofdegreeswhereallmatterwouldbeintheplasmastate.Itisexpectedthatitwillbepossibleinthef uturetocontrolandmaintainaplasmaby externally applied electric and magnetic fields even at these extreme conditions. Themost successful experiment today is the “Tokamak”, a toroidal device where the plasmacan be confined by a combination of externally applied magnetic field in addition to thosegenerated by currents in the plasma itself. The controlled use of fusion energy may solveourenergydemandsforallforeseeablefuture.

Spacesciences: A study of the Earths ionosphere and magnetosphere provides an understanding whichcan be applied to many magnetized planets in or outside our solar system. Although theEarthmaybeuniqueinabiologicalcontext, ithasmanyfeaturesincommonwithmostoftheotherplanetsinoursolarsystem.Asanexample,itwaswellknownthatJupiterw asa significant source of radio noise in our near space environment, but is was eventuallyrealized that also the Earth is a very similar emitter of radio noise, the so called auroralkilometric radiation, which has subsequently been studied extensively by instrumentedspacecrafts.

III. Solitary Waves andSolitons

Manytypeofnonlinearwavesareseeninthespaceplasmas.Asolitarywaveisahumpor dip shaped nonlinear wave of permanent profile.It arises because of the interplaybetween the effects of the nonlinearity and the dispersion (when the effect of dissipa-tion is negligible compared to those of the nonlinearity and dispersion). However, whenthe dissipative effect is comparable to or more dominant than the dispersive effect, oneencountersshockwaves.

Solitonsareaspecifictypeofsolitarywaveswiththeremarkablefeaturethat,whentwo (or more) of them collide, they do not scatter but emerge with the same shape andvelocity.The word ‘soliton’ was coined by Zabusky and Kruskal (1965) after ‘photon’,‘proton’, etc. to emphasize that a soliton is a localized entity which may keep its identityafter an interaction. In the absence of nonlinearity, dispersion can destroy a solitarywave as the various components of the wave propagate at different velocities. Introducingnonlinearity without dispersion again rules out the possibility of solitary waves becausethe pulse energy is continuously injected into high frequency modes.But with bothdispersionandnonlinearity,solitarywavescanagainform.

The history of solitons is an interesting one (Allen, 1998), with solitons first being seenas water waves in canals in England (Russell, 1845).By studying the nature of waves,Russelclaimedthatthepropagationofisolatedwave,wasaconsequenceofthepropertyof the medium rather than the circumstances of the wave generation. Since then it tookrather a long time to establish that some special nonlinear wave equations admit solutions consisting of isolated wave that canpropagate and undergocollisionswithoutloosingtheirrespective identities. The first theoretical work describing solitons was done by Rayleigh(1879), and in 1895 Korteweg and de Vries found the first equation describing a solitarywave (the KdV equation).It was found that the solitary wave appeared as a specialsolutionoftheKdVequation.

IV. IonAcousticWaves

Webeginwiththefluidequationsofcontinuityequation,andmomentumequation

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv)=0 \qquad (2)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=-\frac{q\partial\varphi}{m\partial x}-\frac{1\partial p}{nm\partial x} \qquad (3)$$

andpoissonequationis

$$\frac{\partial^2\varphi}{\partial x^2}=-\frac{e}{E_o}(n_i+n_h) \qquad (4)$$

for electron , me is taken to be so small that we can neglect the left hand side of the momentum equation andthenlettingpe=neKTe(assumedisothermalbecauseofthehighelectronmobility)thenweobtaintheequation

$$\frac{e}{KTe}\frac{\partial\varphi}{\partial x}=\frac{1}{n_e}\frac{\partial n_e}{\partial x} \qquad (5)$$

withsolution

$$n_e=n_0e^{e\varphi/kT_e} \qquad (6)$$

Assumingnowawavelikesolutionsothatallfunctionmaybewrittenasafunctionof ξ=z−Ut,then∂/∂t→−U∂/∂ξ
So that the continuityequofionsbecomes−Uni+niv+ ni=0andintegratingni(n − U)=constant where the prime denotes the derivative w.r.t argument and the momentum equation becomes −Uv+vv=e/miΦandintegrating−Uv+1/2v2=-e/miΦ+constant which is the conservation energy and we set thislastconstantiszero.Solvingforv,wefind

$$v=U\pm[U^2-2e/m_i\Phi]^{1/2}$$

Sothattheiondensityisgivenby

$$n_i=n_0/\sqrt{1-\frac{2e\Phi}{m_iU^2}}$$

andPoissonsequationgives

$$\frac{d^2\phi}{d\xi^2}=-\frac{n_0e}{\epsilon_o}\left[\frac{n_0}{\sqrt{1-\frac{2e\Phi}{m_iU^2}}}-e^{e\phi/kT_e}\right] \qquad (7)$$

Thenusingthechangeofvariableη=dφ/dξwemaywrite

$$\frac{d^2\phi}{d\xi^2}=\eta\frac{d\eta}{d\phi}=\frac{d}{d\phi}\left(\frac{\eta^2}{2}\right) \qquad (8)$$

wherenowwecanseperatevariablesandintegrate,withtheresult

$$\frac{\eta^2}{2}=\frac{n_0}{\epsilon_0}\left[m_iU^2\left(\sqrt{1-\frac{2e\Phi}{m_iU^2}}-1\right)+k\tau_e(e^{e\phi/k\tau_e}-1)\right] \qquad (9)$$

Where the constant terms are chosen so that η→0 as Φ→0. Now solitary wave solution do not exit for all valuesΦandU.Itisclearfromequ8thateΦlessorequal m_iU^2 for a meaningful solution.

If we consider v(Φ)= - η2/2 to be a Pseudopotential with Φ be coordinate and ξ thetime then equation 8 has the form of an equation of motion for a particle moving in aPseudopotentialwell

$$\frac{d^2\varphi}{d\xi^2} = -\frac{dv}{d\varphi} \quad (10)$$

V. Arbitrary amplitude ion acoustic solitary waves in an un-magnetized twoelectronpopulationultra-relativisticdenseplasmas

Introduction: Plasma exhibits a great variety of linear and nonlinear wave phenomena.1,2Ion acousticsolitarywaves(IASWs)areoneoftheimportantnonlinearphenomenainplasmasystems.These waves have been studied both theoretically and experimentally.3−9The study ofsolitary waves in a plasma having two temperature electrons has already appeared inliterature.10Thepresenceofanexternalsourcecausesacertainnumberofelectronstobe heated preferentially in plasmas. Due to such heating the electrons can have differenttemperature such as its existence in auroral latitude where the phenomena of heating andinjection can give rise to the existence of hot and cold electrons in plasma.A complexplasma with two electron temperature has been found in the noctilucent cloud region ofthe Earth's atmosphere where the energetic particles precipitation affects the mesospherecharge balance.11Two electron temperature plasma can also be seen due to an isotropic electron distributionsintwoperpendiculardirections.Theauroralplasmaisobservedto support the propagation of ion acoustic solitary and double layer structures in thepresence of hot and cold electrons.12Mahmood et al.13investigated arbitrary amplitudeion acoustic solitary waves in the presence of adiabatically heated ions and immobile dustin magnetized plasmas.Recently, Shah et al.14studied the propagation of ion acousticsolitons in unmagnetized inhomogeneous multi ion component plasmas with vortex dis-tributed electrons. Some authors have studied IASWs and double layers in a relativisticplasma.15,16Energetic electron distributions are observed in the different regions of themagnetosphere.In hot places as planet interiors and white dwarfs,17dense plasma arecharacterized by high densities and low temperatures. The quantum effects in collectivebehavior of a plasma system becomes important when the inter-particle distances are comparable or less than the deBroglie thermal wavelength λB=h/(2πmekBT)1/2orequivalently when the thermal energies of plasma species are less than Fermi-energies.18In such cases the plasma becomes degenerate, in which the plasma ingredients are under effective influence of Pauliexclusionprincipleandclassicalstatisticalassumptionsbreakdown. For a cold neutron star the densities can be as high as 1015gm/cm−3in the core,which is several times the density of an atomic nuclei. In extreme conditions such as themiddleofasupernovaorthecoreofamassivewhitedwarfthedensitiescanbeevencatas-trophically higher. At these very high densities the electrons and positrons may becomeultra-relativistic giving rise to the collapseofstarunderitsgiantgravitationalforce.19,20In quantum plasmas, two population electrons, i.e., densely and sparsely populated elec-trons can exist because, according to the Fermi gas model, Fermi temperature is directly relatedtonumberdensityoffermionicparticles.ThemathematicalrelationisgivenasTFej=(3π2nj0)2/3h¯2/(2kBme)(j=c, hisforcoldandhotelectrons),TFeistheelectron Fermi temperature, h¯ is Planck's constant dividedby2π,nj0istheequilibriumelectron density, kBis the Boltzmann constant, and meis the electron mass. Such a two electronpopulation can be found in laser-produced plasmas21−24as well as in dense astrophysicalplasmas.25,26The plasma in the interior of white dwarfs and in the crust of neutron starsis extremely dense and highly degenerate with electron number densities nj0>1020cm−3 and Fermitemperature lying in the range105K<TFe<108K.27,28Atsuchhighdensities,the electron Fermi temperature is usually greater than the electron thermal temperature.The equation of state for degenerate electrons in interstellar compact objects (e.g., whitedwarfs, neutron stars) is mathematically explained by Chandrasekhar29for two limits,namely,nonrelativisticandultrarelativisticlimits. Thedegenerateelectronequationof stateof Chandra sekhar is Pe n5/3 forthenonrelativisticlimitand Pe n4/3forthe ultrarelativistic limit, where Peis the degenerate electron pressure and neis the degen-erate electron number density.Recently, Mendonca and Shukla30studied ion acousticwaves in a ultra-cold neutral plasma. Rasheed et al.31studied ion acoustic solitary wavesin ultra-relativistic degenerate pair-ion plasmas.Chandra and Ghosh32examined theo-retically modulational instability of electron-acoustic waves in relativistically degeneratequantum plasma. More recently, Mamun and Shukla33investigated solitary waves in anultra-relativisticdegeneratedensedustyplasma.

Again double layers occur naturally inavariety of space plasma environments, suchas auroras, solar wind, extragalactic jets, etc. Double layers can accelerate, decelerate,or reflect the plasma particles. The formation of double layers has been received a greatdealofinterest34−40because of its relevance in cosmic plasmasandinplasmathrustsfor space properties of plasma.It has been argued that small amplitude double layers mayaccount for a large portion of the total potential on auroral field lines and may explain thefine structure of auroral kilometric radiation. However, to the best of our knowledge, thestudy of the IASWs and double layers in an ultra-relativistic degenerate plasma with coldand hot electrons have not yet been done.The aim of our present paper is therefore toelucidate the the propagation of arbitrary amplitude IASWs and double layers in a plasmahaving cold and hot electron fluid, and inertial ultra-cold ions. The model is relevant tocompact interstellar objects, particularly to white dwarfs.To study IASWs in such aplasma system, Sagdeev's41pseudopotential approach has been used. Here we study theIASWs and small amplitude double layers in an unmagnetized three component plasmaconsistingofacoldandhotelectronsandinertialionsinultra-relativistic degenerate denseplasma. The organization of the paper isas follows. InsectionII thebasics etofequationsare given and the linear dispersion relation were derived for ionacoustic solitary waves. InsectionIII we derived the Sagdeev's pseudo potential and small amplitudedoublelayersolution.SectionIViskeptforresults,anddiscussionwhilesectionVis kept for conclusion.

Governing equations andlinear waves: We consider a homogeneous, collisionless, unmagnetized ultra-relativistic degenerate denseplasma consisting of a cold and hot electron fluid and inertial ultra-cold ions.In equi-librium,we have nc0+ nh0=Zini0,where the subscript "0" stands for unperturbedquantities,andZirepresentstheionchargestate.Thebasicsystemofequationsforone dimensional propagation of nonlinear ion acoustic waves in such a plasma model isgovernedby[33].

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \qquad (11)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{Z_i e}{m_i}\frac{\partial \phi}{\partial x} \qquad (12)$$

$$\frac{1}{e}\frac{\partial P_c}{\partial x} = n_c \frac{\partial \phi}{\partial x} \qquad (13)$$

$$\frac{1}{e}\frac{\partial P_h}{\partial x} = n_h \frac{\partial \phi}{\partial x} \qquad (14)$$

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(n_c + n_h - Z_i n_i) \qquad (15)$$

Equations (1)-(5) in normalized form in one dimension (1D) are

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \qquad (16)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x} \qquad (17)$$

$$\frac{3\beta_1}{4n_c}\frac{\partial n_c^{4/3}}{\partial x} = \frac{\partial \phi}{\partial x} \qquad (18)$$

$$\frac{3\beta_2}{4n_h}\frac{\partial n_h^{4/3}}{\partial x} = \frac{\partial \phi}{\partial x} \qquad (19)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{1+\alpha} n_c + \frac{\alpha}{1+\alpha} n_h - n_i \qquad (20)$$

In the above equations, niand nc(nh), is the ion and cold (hot) electron number densitynormalized to its equilibrium value ni0andnc0(nh0), viis the ion velocity normalizedto $C_i = \sqrt{Z_i m_e c^2/m_i}$is the ion mass, and c is the speed of light in vacuum, α =nh0/nc0.φistheelectrostaticwavepotentialnormalizedtoZimec2/ewithedenotingtheelementary charge, β1= K0δ, and β2= α1/3β1, K0= (72π)−1/3≡ 0.164102, δ = λcn1/3,andλc=h¯/mec=2.425×10−10cm.isthereducedComptonwavelengthandwehave usedtheequationofstateforPc=(3/π)1/3(h¯c/8)n4/3andPh=(3/π)1/3(h¯c/8)n4/3for ultra- c h co relativisticdegenerateelectronsindenseplasmas.Timeandspacevariablesarenormalizedrespectivelytotheinverseofion plasmafrequency$\omega_{pi}^{-1} = m_i/4\pi n_{i0} Z_i^2 e^2$

andtheDebyelength,$\lambda_{Di} = \sqrt{m_e c^2/4\pi n_{i0} c^2}$~~respectively.~~ Herewehaveassumedthat Z=1.

Now, we study the dispersion properties of linear waves, for which we use the linearperturbationsofdependentnormalizedquantitiessuchasni=1 +Eni1,nc=1 + Enc1,nh= 1 + Enh1, vi= Evi1, and φ = Eφ1, where E is a small parameter, in the set of equations(6)-(10).Hencethenormalizedequationsare

$$\frac{\partial n_{i1}}{\partial t} + \frac{\partial v_{i1}}{\partial x} = 0 \qquad (21)$$

$$\frac{\partial v_{i1}}{\partial t} = -\frac{\partial \varphi_1}{\partial x} \quad (22)$$

$$\beta_1 \frac{\partial n_{c1}}{\partial x} = \frac{\partial \varphi_1}{\partial x} \quad (23)$$

$$\beta_2 \frac{\partial n_{h1}}{\partial x} = \frac{\partial \varphi_1}{\partial x} \quad (24)$$

$$\frac{\partial^2 \varphi_1}{\partial x^2} = \frac{1}{1+\alpha} n_{c1} + \frac{\alpha}{1+\alpha} n_{h1} - n_{i1} \quad (25)$$

Weassumethatalltheperturbedquantitiesareproportionaltoexp[i(kx−ωt)],wherekisthewavenumberandωisthefrequency.Thusweobtainthedispersionrelationas

$$\omega^2 = \frac{k^2}{k^2 + \frac{\beta_2 + \alpha\beta_1}{\beta_1\beta_2(1+\alpha)}} \quad (26)$$

ThepropertiesoflinearionacousticwavesaredisplayedinFig.1forseveralvaluesof nc0.It shows that an increase in density nc0enhances the frequency of the electronacoustic wave for a range of wave number, k, values. Also, it is found that a system withhigherdensityachievestheasymptoticvalueω=1earlierthantheonewithlowerdensity

Pseudopotential approach: To obtain a travelling wave solution we make all the dependent variables depend on asingle independent variable $\xi = x - Mt$, where M is the Mach number i.e., the velocityofsolitarywave.Nowequations(6)-(10)canbewrittenas

$$-M\frac{dn_i}{d\xi} + \frac{d}{d\xi}(n_i v_i) = 0, \quad (27)$$

$$-M\frac{dv_i}{d\xi} + v_i\frac{dv_i}{d\xi} = -\frac{d\phi}{d\xi}, \quad (28)$$

$$\frac{3\beta_1}{4n_c}\frac{dn_c^{4/3}}{d\xi} = \frac{d\phi}{d\xi} \quad (29)$$

$$\frac{3\beta_2}{4n_h}\frac{dn_h^{4/3}}{d\xi} = \frac{d\phi}{d\xi} \quad (30)$$

$$\frac{d^2\phi}{d\xi^2} = \frac{1}{1+\alpha}n_c + \frac{\alpha}{1+\alpha}n_h - n_i. \quad (31)$$

Integratingequations(17)-(20)andusingtheboundaryconditions:
$v_i \to 0$, $n_i \to 1$, $n_c \to 1$, $n_h \to 1$, $\varphi \to 0$ as$\xi \to \pm\infty$wewrite(aftersomesimplealgebra)

$$n_i = \frac{1}{\sqrt{1 - \frac{2\phi}{M^2}}}, \quad (32)$$

$$n_c = \left(1 + \frac{\phi}{3\beta_1}\right)^3 \quad (33)$$

$$n_h = \left(1 + \frac{\phi}{3\beta_2}\right)^3. \quad (34)$$

Toobtainthepseudopotentialψ(φ),wenoticethatequation(21)canbeexpressedaswiththehelpof(22-24)
whereψ(φ),thepseudopotential,isgivenby

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \psi(\phi) = 0 \quad (35)$$

$$\psi(\phi) = M^2\left(1-\sqrt{1-\frac{2\phi}{M^2}}\right)+\frac{3\beta_1}{4(1+\alpha)}\left\{1-\left(1+\frac{\phi}{3\beta_1}\right)^4\right\}+\frac{3\alpha\beta_2}{4(1+\alpha)}\left\{1-\left(1+\frac{\phi}{3\beta_2}\right)^4\right\} \tag{36}$$

Inderivingequation(26),thefollowingboundaryconditionswereused.ni→1,nc→1, nh→1,vi→0,φ→0anddφ→0atξ→±∞ $d\xi$
Forsolitarysolutionstoexist,thefollowingconditionsmustbesatisfied:

$$\text{i)}\ \psi(\phi) = 0, \left(\frac{d\psi}{d\phi}\right)_{\phi=0} = 0 \text{ and} \left(\frac{d^2\psi}{d\phi^2}\right)_{\phi=0} < 0,$$

and ii) $\psi(\phi) < 0$for ϕlying between 0 and ϕm, i.e., either for 0 <ϕ<ϕm (compressive) or ϕm <ϕ<0 (rarefactive)
Againforthedoublelayersolutionsthefollowingconditionsmustbesatisfied.

(i) $\psi(\phi) = 0$ at $\phi = 0$ and $\phi = \phi_m$,

(ii) $\left(\frac{d\psi}{d\phi}\right)_{\phi=0} = 0$ and $\left(\frac{d\psi}{d\phi}\right)_{\phi=\phi_m} = 0$

(iii) $\left(\frac{d^2\psi}{d\phi^2}\right)_{\phi=0} < 0$, and $\left(\frac{d^2\psi}{d\phi^2}\right)_{\phi=\phi_m} < 0$,

Thecondition(i)fortheexistenceoflocalizedIASWsrequiresthemachnumbertosatisfy

$$M > \sqrt{\frac{\beta_1\beta_2(1+\alpha)}{\beta_2+\alpha\beta_1}} \tag{37}$$

This represents the lower limit of Mach number for existence of solitons. The upper M^2 limit of M, Mmax, can be obtained by the condition ψ(φc) >0, where is the maximum value of ϕ for which the ion density is real

$$\phi_c = \frac{M_{max}^2}{2}$$

Inthesmallamplitudelimit,φ«1,theequation(26)canbewrittenas

$$\psi(\varphi)=A_1\varphi^2+A_2\varphi^3+A_3\varphi^4+O(\varphi^5) \tag{38}$$

Where
Now,usingthefirsttwoboundaryconditionsfordoublelayers,wehave2φm=−A_2/$A_3$and the

$$A_1 = \frac{1}{2M^2} - \frac{\beta_2+\alpha\beta_1}{2\beta_1\beta_2(1+\alpha)}$$

$$A_2 = \frac{1}{2M^4} - \frac{\beta_2^2+\alpha\beta_1^2}{9\beta_1^2\beta_2^2(1+\alpha)}$$

$$A_3 = \frac{5}{8M^6} - \frac{\beta_2^3+\alpha\beta_1^3}{108\beta_1^3\beta_2^3(1+\alpha)}$$

Sagdeev's potential ψ(φ) is given by ψ(φ) = A3φ2(φm− φ)2.The double layersolutioncanbeobtainedas[42]

$$\phi = \frac{\phi_m}{2}\left[1 - tanh\left(\frac{2\xi}{\Delta}\right)\right] \tag{39}$$

Where

$$\Delta = \frac{\sqrt{-8/A_3}}{|\phi_m|}.$$

Representsthewidthofthedoublelayerprovided A_3<0Itisto be noted from equation(29) that the nature of the double layerdependsuponthesignofA2, i.e for A2>0 a compressive double layer exists, whereas for A2<0 we would have ararefactivedoublelayer.

VI. Results and Discussion

In this section, we comprehend the propagation of IASWs in an ultra-relativistic degen-erate dense plasma. Numerical studies have been made using typical plasma parameterscorresponding to white dwarfs17,33. We have investigated the impact of different plasmaparameters such as β1or β2that corresponds to different density regimes, the densityratioofdegenerateelectrons(α=nh0/nc0)etc.Forarbitraryamplitudesolitarywaves, in Fig.2 we have plotted ψ(φ)againstφfordifferentvaluesofdensitync0.Itisseenthat amplitude and width of solitary waves decreases with increase of nc0. It is also seenthatsolitonsceasetoexistwhennc0crossesacertainlimit.Itmaybementionedthatthis limit of course depends on the other parameter. Fig.3 shows the pseudo potential ψ(φ) as a function of φf or differentvaluesofM.Hereitisobservedthatthepoten-tial well depth of the Sagdeev's potential curve increases as the Mach number increases,implying that faster pulse excitations will be taller and there exist a critical value of Mbeyond which the solitary waves cease to exist. Fig.4 shows the formation of the poten-tial wells in the positive φ-axis, which corresponds to the formation of the IASWs with positive potential for different valuesofα.Itisdepictedthattheamplitudeandwidthof the solitary wave structures decreases with the increase of α.This means that theamplitude and width of the solitary waves in the medium decreases when the hot-to-coldelectronnumberdensityincreases.InFig.5wehaveplottedψ(φ)[Eq.(29)]againstφfor several values of nc0.The figure shows that the amplitude of the negative potentialdouble layer increases with increasing nc0. It is seen that as nc0decreases, the negativepotential double layer shrinks. Fig.6 depicts the double layer solution for different valuesof nc0.It turnsout that the nature of ion acoustic doublelayer depends sensitively on n_{c0}.

VII.Conclusion

Rosuvastatin 20 mg on every other regimen had equal effect when compared to daily dose regimen of atorvastatin 40 mg &rosuvastatin 20mg.

A study of linear and nonlinear IASWs in an ultra-relativistic degenerate dense plasmaconsisting of a cold and hot electron fluid and inertial ultra-cold ions was carried out. Inview of the standard normal-mode analysis we derived the dispersion relation of IASWs.Sagdeev's pseudopotential approach has been used to find exact large amplitude solitarywave solutions.Numerical investigations are conducted to see the effect of parameterslike α, β1(α n1/3), M on the existence of solitons. It is found that the nature of IASWsdepend upon relevant plasma parameters. Also it is found that the small amplitude dou-ble layers can exist in such a plasma system. Numerical results reveal that the width andamplitude of the ion acoustic double layers are significantly affected by ultra-relativistic degenerate electrons.

References

[1]. T.H.Stix,Wavesinplasma(AmericanInstituteofPhysics,Newyork,1992)P.25.
[2]. F.Verheest,WavesinDustySpaceplasma(Kluwer,Dordrecht,2000)P.18.
[3]. H.Ikeji,R.J.TaylorandD.R.Baker,Phys.Rev.Lett.25,11(1970).
[4]. R.A.Cairns,A.A.Mamun,R.Bingham,andP.K.Shukla,Phys.Scr.T63,80(1996).
[5]. K.YoshimuraandS.Watanabe.J.Phys.Soc.Jpn.60,82(1991).
[6]. V.V.Konotop,Phys.Rev.E53,2843(1996).
[7]. HashimotoandOno,J.Phys.Soc.Jpn.33,605(1972).
[8]. P.Eslami,M.MottaghizadehandH.R.Pakzad,Phys.Scr.83,065502(2011).
[9]. S.MahmoodandH.Saleem,Phys.Plasmas.9,724(2002).
[10]. K.H.IbohanbiSinghandG.C.Das,IEEETrans.PlasmaSci.17,863(1989).
[11]. O.Havnes,ProceedingsoftheICTPSummerCollegeonPlasma,Trieste,Italy,28July-24August,2007.
[12]. R.V.ReddyandG.S.Lakhina,Planet.SpaceSci..39,1343(1991).
[13]. S.MahmoodandS.Hussain,Phys.Plasmas.14,082303(2007).
[14]. A.Shah,S.MahmoodandQ.Haque,Phys.Plasmas.17,112320(2010).
[15]. S.K.El-Labany,H.O.NafieandA.El-Sheikh,J.PlasmaPhys.56,13(1996).
[16]. Y.Nejoh,J.PlasmaPhys.,37,487(1987).
[17]. G.Manfredi,FieldsInst.Commun.46,263(2005).
[18]. M.Bonitz,D.Semkat,A.Filinov,V.Golubnychyi,D.Kremp,D.O.Gericke,M.S.Murillo,V.Filinov,V.Fortov,W.Hoyer,J.Phys.A,36,5921(2003)
[19]. S.Chandrasekhar,AnIntroductiontotheStudyofStellarStructure,Chicago,Ill.,(TheUniversityofChicagopress),(1939).
[20]. S.L.Shapiro,andS.A.Teukolsky,BlackHoles,WhiteDwarfsandNeutronStars,(Wiley,NewYork),(1983).
[21]. D.Kremp,T.Bornath,M.Bonitz,andM.Shlanges,Phys.Rev.E,60,4725(1999).
[22]. A.V.Andreev,JETPLett.,72,238(2000).
[23]. R.Bingham,J.T.Mendonca,andP.K.Shukla,PlasmaPhys.ControlledFusion,46,R1(2004).
[24]. W.M.Moslem,P.K.Shukla,S.Ali,andR.Schlickeiser,Phys.Plasmas,14,042107(2007).
[25]. G.Chabrier,F.Douchin,andY.Potekhin,J.Phys.Condens.Matter,14,133(2002).
[26]. Y.F.Jung,Phys.Plasmas,8,3842(2001).
[27]. V.R.Pandharipande,D.Pines,andR.A.Smith,Astrophys.J,208,550(1976).
[28]. G.Chabrier,D.Saumon,andA.Y.Potekhin,J.Phys.A,39,4411(2006).
[29]. S.Chandrasekhar,Mon.Not.R.Astron.Soc.,170,405(1935);Astrophys.J,74,81(1931).
[30]. J.T.MendoncaandP.K.Shukla,Phys.Plasmas,18,042101(2011).
[31]. A.Rasheed,N.L.Tsintsadze,andG.Murtaza,Phys.Plasmas18,112701(2011).
[32]. S.Chandra,andB.Ghosh,Astrophys.SpaceSci.,342,417(2012).

[33]. A.A.MamunandP.K.Shukla,Phys.Plasmas,17,104504(2010);Phys.Lett.A,374,4238(2010).
[34]. M.Temerin,K.Cerny,W.Lotko,andF.Mozer,Phys.Rev.Lett.48,1175(1982).
[35]. Sk.A.Islam,A.BandyopadhyayandK.P.Das,Phys.Scr.82,045505(2010).
[36]. M.A.Raadu,Phys.Rep.178,25(1989).
[37]. M.K.Mishra,A.K.Arora,andR.S.Chhabra,Phys.Rev.E66,046402(2002).
[38]. T.Cattaert,F.Verheest,andM.A.Hellberg,Phys.Plasmas12,042901(2005).
[39]. S.LiuandJ.Liao,Phys.Scr.82,045902(2010).
[40]. R.AmourandM.Tribeche,Phys.Lett.A373,1951(2009).
[41]. R.Z.Sagdeev,Rev.PlasmaPhysics4,23(1966).
[42]. L.A.Gougam,andM.Tribeche,Astrophys.SpaceSci.331,181(2011).