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Super Cube Root Cube Mean Labeling of Graphs

Radhika V.S.[1] Vijayan A.[2]

[1](*Research Scholar, Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Kanyakumari District, Tamil Nadu, India.*)

[2](*Research Scholar, Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Kanyakumari District, Tamil Nadu, India.*)

Abstract: *A function f is called super cube root cube mean labeling of a graph G = (V,E) with p- vertices and q- edges if f: V(G) →{ 1, 2, . . ., p + q} is injective and the induced function* f^* *defined as* $f^*(uv) = \left\lfloor \sqrt[3]{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$ *or* $\left\lceil \sqrt[3]{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil$. *For all* $uv \in E(G)$ *is bijective. Then the resulting edge labels are distinct. A graph that admits a super cube root cube mean labeling f is called a super cube root cube means graph. In this paper we introduce super cube root cube mean labeling and investigate super cube root cube mean labeling of path* P_n*, Comb graph, ladder graph, pendant vertex attached with comb graph, pendant vertex attached with the ladder graph,* $C_n \odot K_1$*,* $P_n \odot K_3$ *and Fish graph.*

Keywords: *cube, root, square, graph*

1. INTRODUCTION

By a graph G = (V,E) we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling was refer to J.A. Gallian [1]. For all other standard terminology and notations we follow F.Harray [2]. The concept of Mean Labeling has been introduced by S. Somasundaram and R. Ponraj [7]. Root square Mean labeling of Graphs has been introduced by S. Sandhya, S. Somasundaram and A. Anusa[4].Root cube mean labeling of graphs has been introduced by Gowri and Vembarari[6]. Cube root cube Mean labeling was introduced by S. Kulandhai Therese and K. Romila [5]. Motivated from the above works, we introduced a new type of labeling called Super Cube root cube Mean Labeling of graphs.

In this paper, we investigate the super cube root cube mean labeling of some graphs such as Path P_n, Comb graph, ladder graph, pendant vertex attached with comb graph, pendent vertex attached with the ladder graph, $C_n \odot K_1$, $P_n \odot K_3$ and Fish graph. We now give the following definitions which are useful for the present investigation.

Definition 1.1. A function f is called a cube root cube mean labeling of a graph G, if f : V(G) → { 1, 2, …, q + 1 } is injective and the induced edge function f*: E(G) → { 1, 2, . . ., q} defined as f*(e= uv) = $\left\lfloor \sqrt[3]{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$ or $\left\lceil \sqrt[3]{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil$ is bijective. Thus a graph which admits cube root cube mean labeling is called as cube root cube mean graph.

Definition 1.2. Let f : V(G) → {1,2,. . ., p +q} be an injective function. For a vertex labeling f, the induced edge labeling f*(e = uv) is defined by f*(e) $\left\lfloor \sqrt[3]{\frac{f(u)^3 + f(v)^3}{2}} \right\rfloor$ or $\left\lceil \sqrt[3]{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil$. Then f is called a super cube root cube mean labeling if f(V(G)) ∪{f(e) : e∈E(G)} = { 1,2, . . ., p +q}. A graph which admits super cube root cube mean labeling is called a super cube root cube mean graph.

Definition 1.3. A walk in which vertices are distinct is called a path. A path on n- vertices is denoted by P_n.

Definition 1.4. The graph obtained by joining a single pendent edge to each vertex of a path is called as comb graph.

Definition 1.5. The product graph $P_2 \times P_n$ is called as Ladder and is denoted by L_n.

II . MAIN RESULTS

Theorem 2.1. Any path P_n is a super cube root cube mean labeling graph

Proof . Let G = P_n be the path with vertices $u_1, u_2, \ldots, u_n$.

Here , p + q = 2n-1.

Define a function $f : V(G) \rightarrow \{1,2, \ldots, p + q\}$ by
$f(u_i) = 2i-1, 1 \leq i \leq n$.
The edges are labeled with
$f^*(u_iu_{i+1}) = 2i, \; 1 \leq i \leq n-1$.
Thus, the edge labels are distinct. Hence, any path P_n is a super cube root cube mean graph.
Example 2.2. A super cube root cube mean labeling of P_6 is shown below:

Figure :1

Theorem 2.3. Any cycle C_n is a super cube root cube mean graph.
Proof . Let $G = C_n$ be the cycle with vertices $u_1, u_3, \ldots, u_n$.
Here, $P + q = 2n$.
Define a function $f : V(G) \rightarrow \{1,2, \ldots, p + q\}$ by,

$$f(u_i) = \begin{cases} 2i-1, 1 \leq i \leq n-2 \\ 2i, n-1 \leq i \leq n \end{cases}.$$

Then, we get distinct edge labels.
Hence, $f(V(G) \cup \{f(e) : e \in E(G)\} = \{1,2, \ldots, p + q\}$. Hence C_n is a super cube root cube mean.
Example 2.3. Cube Root cube mean labeling of C_7 is shown below

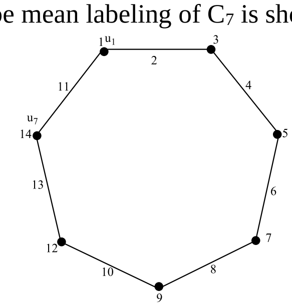

Figure :2

Theorem 2.5. The Ladder L_n is a super cube root cube mean graph.
Proof . Let $L_n = P_n \times P_2$ be the ladder graph.
Let u_i and v_i , $1 \leq i \leq n$ be its vertices. Let it be denoted by G.The graph is displayed below:

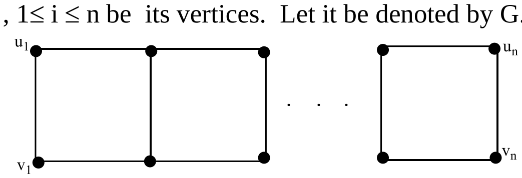

Figure : 3

Here $P + q = 5n - 2$.
Define a function $f : V(G) \rightarrow \{1,2, \ldots, p + q\}$ by
$f(u_1) = 1$,
$f(u_2) = 3$,
$f(u_i) = 5i-5, 3 \leq i \leq n$,
$f(v_1) = 5$,
$f(v_2) = 8$,
$f(v_i) = 5i-1, 3 \leq i \leq n-1$,
$f(v_n) = 5n-2$.
The edges are labeled with
$f^*(u_1u_2) = 2$,
$f^*(u_2u_3) = 9$,
$f^*(u_iu_{i+1}) = 5i-2, 3 \leq i \leq n-1$,
$f^*(v_1v_2) = 7$,
$f^*(v_i,v_{i+1}) = 5i+1, 2 \leq i \leq n-1$,
$f^*(u_1v_1) = 4$,

$f^*(u_1v_2) = 6$,
$f^*(u_iv_i) = 5i - 3,\ 3 \le i \le n$.
Thus, the edge labels are distinct.
Thus, $f(V(G)) \cup \{f(e) : e \in E(G)\} = \{1, 2, \ldots, P + q\}$.
Hence L_n is a super cube root cube mean graph

Example 2.6. A super cube root cube mean labeling of L_7 is displayed below :

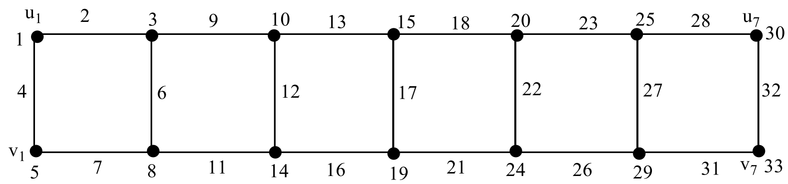

Figure :4

Theorem 2.7. Comb graph is a super cube root cube mean graph .

Proof . Consider the path $P_n = u_1u_2 \ldots u_n$.
The comb graph is obtained from a path P_n by joining the vertex v_i to u_i, $1 \le i \le n$. Let it be denoted by $P_n \odot k_1$. Let $G = P_n \odot k_1$. The graph is displayed below :

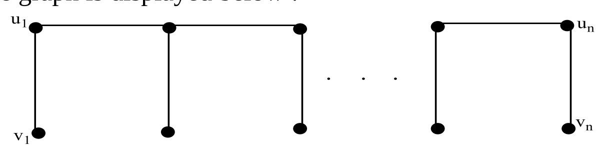

Figure :5

Here $p + q = 4n-1$.
Define a function $f : V(G) \to \{1,2,\ldots, p+q\}$ by
$f(u_1) = 3$,
$f(u_i) = 4i - 3,\ 2 \le i \le n$,
$f(v_1) = 1$,
$f(v_i) = 4i,\ 2 \le i \le n-1$,
$f(v_n) = 4n-1$.
The edges are labeled with $f^*(u_1,u_2) = 5$,
$f^*(u_iu_{i+1}) = 4i-1,\ 2 \le i \le n-1$,
$f^*(u_iv_i) = 4i-2,\ 1 \le i \le n$.
Thus, the edge labels are distinct.
Hence $f(V(G)) \cup \{f(e) : e \in E(G)\} = \{1,2,\ldots, p+q\}$.
Hence, $P_n \odot K_1$ is a super cube root cube mean labeling graph.

Example 2.8. Super cube root cube mean labeling of $P_6 \odot K_1$ is shown below

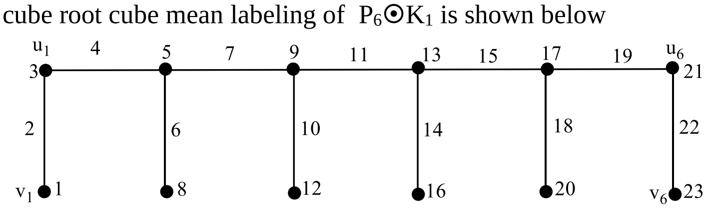

Figure :6

Theorem 2.9. Let G be a graph obtained by joining a pendant vertex with a vertex of degree two on both ends at a comb graph. Then G is super cube root cube mean graph.

Proof. Comb ($P_n \odot K_1$) is a graph obtained from a path $P_n = u_1u_2 \ldots u_n$ by joining a vertex v_i to u_i, $1 \le i \le n$. Let G be a graph obtained by joining the pendant vertex w to u_1 and Z to u_n (a vertex of degree 2).
The graph is displayed below :

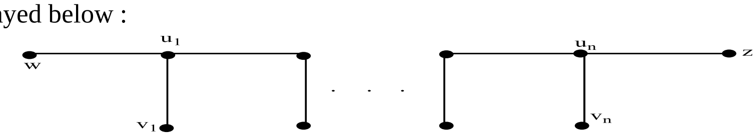

Figure :7

Here p + q = 4 n + 3.
Define a function $f : V(G) \rightarrow \{1,2, \ldots, p+q\}$ by
$f(w) = 1$,
$f(u_i) = 4i-1, \ 1 \leq i \leq n$,
$f(z) = 4n + 3$,
$f(v_i) = 4i + 2, 1 \leq i \leq n$.
The edge labeled with
$f^*(wu_1) = 2$,
$f^*(u_iu_{i+1}) = 4i +1, 1 \leq i \leq n-1$,
$f^*(u_nz) = 4n+1$,
$f^*(u_iv_i) = 4i, 1 \leq i \leq n$.
Thus, the edge label are distinct. Hence, $f(V(G)) \cup \{ f(e) : e \in E(G) = \{1,2, \ldots, p + q\}$.
Hence G is a super cube root cube mean labeling graph.
Example 2.10. The super cube root who mean labeling of G when n = 5 is displayed below :

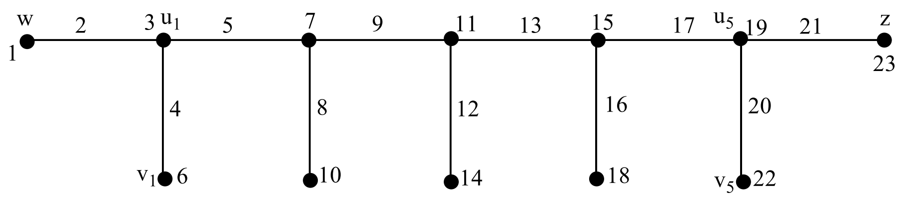

Figure : 8

Theorem 2.1. Let G be a graph obtained by joining a pendant vertex with a vertex of degree two of a comb graph. Then G is a super cube root cube mean graph.

Proof . Comb $(P_n \odot K_1)$ is a graph obtained from a path $P_n = u_1u_2 \ldots u_n$ by joining a vertex v_i to u_i, $1 \leq i \leq n$. Let G be a graph obtained by joining a pendent vertex w to u_n (a vertex of degree 2). The graph is displayed below :

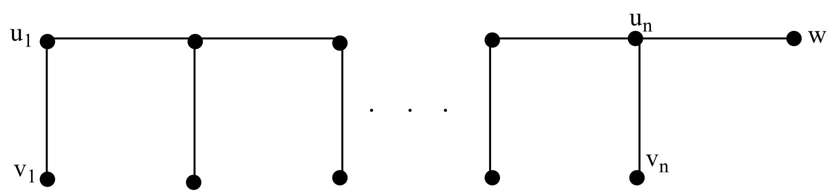

Figure :9

Here P + q = 4n +1.
Define a function $f : V(G) \rightarrow \{1, 2, \ldots, p+q\}$ by
$f(u_1) = 3$,
$f(u_i) = 4i -3, 2 \leq i \leq n$,
$f(w) = 4n+1$,
$f(v_1) = 1$,
$f(v_i) = 4i, 2 \leq i \leq n$.
The edges are labeled with
$f^*(u_1u_2) = 4$,
$f^*(u_iu_{i+1}) = 4i-1, 2 \leq i \leq n-1$.
$f^*(u_nw) = 4n-1$,
$f^*(u_iv_i) = 4i-2, 1 \leq i \leq n$.
Thus, the edge labels are distinct.
Hence, $f(V(G)) \cup \{f(e) : e \in E(G)\} = \{1,2, \ldots, p+q\}$.
Hence G is a Super cube root cube mean labeling graph.
Example 2.12. The super cube root cube mean labeling of G when n = 5 is shown below :

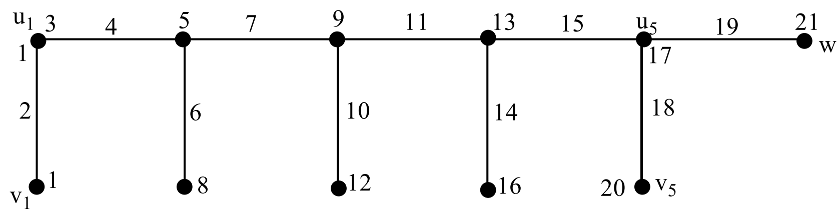

Figure : 10

Theorem 2.13. Let G be a graph obtained from a Ladder L_n, $n \geq 2$ by joining a pendant vertex with a vertex of degree two on both sides of upper and lower path of the ladder. Then G is a super cube root cube mean labeling graph.

Proof . Let $L_n = P_n \times P_2$ be a Ladder. Let G be a graph obtained from a ladder by joining pendant vertices w,z,x,y with u_1, u_n, v_1, v_n (vertices of degree 2) respectively on both sides of upper and lower path of the ladder.The graph is displayed below :

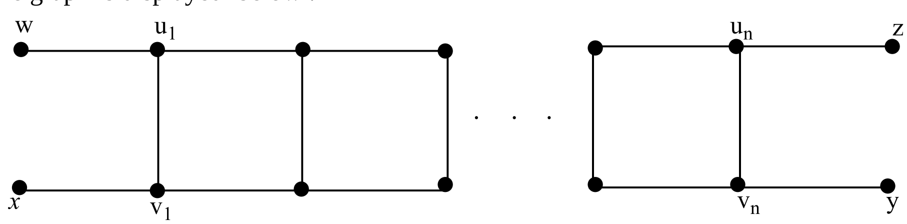

Figure : 11

Here p + q = 5n + 6.

Define a function f : V(G) → {1,2, . . ., p + q} by

$f(x) = 1,$

$$f(u_i) = \begin{cases} 6i-3, 1 \leq i \leq 2 \\ 5i-1, 3 \leq i \leq n \end{cases}$$

$f(w) = 5n + 4,$

$f(y) = 4,$

$f(v_1) = 7,$

$f(v_i) = 5i + 3,\ 2 \leq i \leq n,$

$f(z) = 5n + 6 .$

The edges are labeled with ,

$f^*(xu_1) = 2,$

$f^*(u_1u_2) = 8,$

$f^*(u_iu_{i+1}) = 5i + 2,\ 2 \leq i \leq n-1,$

$f^*(u_nw) = 5n + 2,$

$f^*(yv_1) = 5,$

$f^*(v_iv_{i+1}) = 5i,\ 1 \leq i \leq n-1.$

$f^*(v_nz) = 5n + 5,$

$f^*(u_iv_i) = 5i+1,\ 1 \leq i \leq n.$

Thus , the edge labels are distinct.

Hence, f (V(G))∪{f(e) : e∈E(G) } = { 1,2, . . .,p +q}. Hence G is a super cube root cube mean graph

Example 2.14. The super cube root cube mean labeling of G when n = 5 is displayed below :

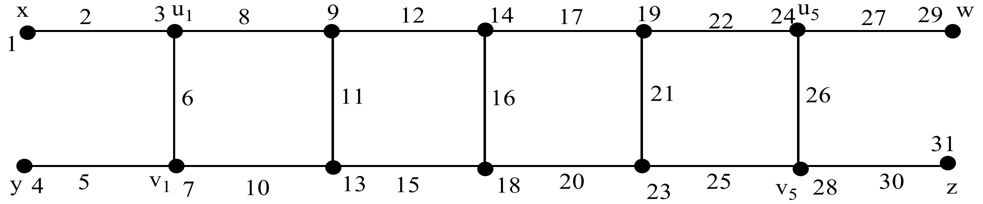

Figure : 12

Theorem 2.15. $C_n \odot K_1$ is a a super cube root cube mean graph.

Proof . Let C_n be the cycle $u_1u_2\ldots u_nu_1$. Let v_i be the pendant vertex attached to $u_i, 1 \leq i \leq n$. Let it be denoted as G. The graph is displayed below :

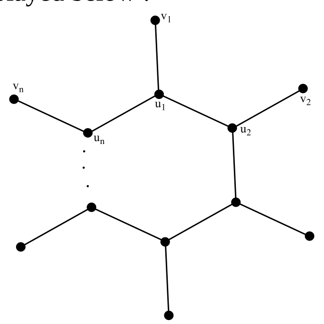

Figure : 13

Here p + q = 4 n.

Define a function $f : V(G) \rightarrow \{1,2, \ldots, p + q\}$ by

$$f(u_1) = 3, f(u_i) = \begin{cases} 4i - 3, & 2 \leq i \leq n-2 \\ 4i - 2, & n-1 \leq i \leq n \end{cases},$$

$f(v_1) = 1$,

$f(v_2) = 8$,

$f(v_i) = 4i$, $3 \leq i \leq n$.

Then, the corresponding edge labels are distinct.

Hence we get $f(V(G)) \cup \{f(e) : e \in E(G)\} = \{1, 2, \ldots, p+q\}$.

Hence $C_n \odot k_1$ is a super cube root cube mean graph.

Example 2.16. The super cube root cube mean labeling $C_n \odot k_1$ is shown below :

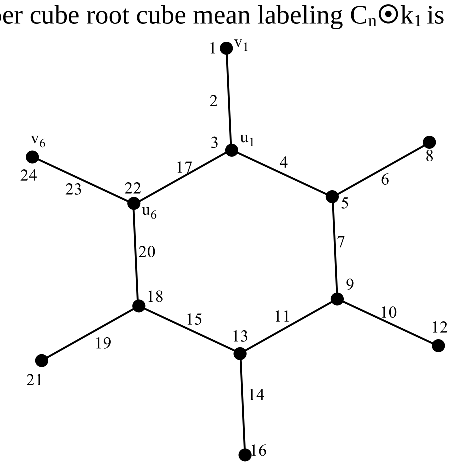

Figure :14

Theorem 2.17. $P_n \odot K_3$ is a super cube root cube mean graph.

Proof :

Let P_n be the path with vertices $u_1, u_2, \ldots, u_n$. Let $v_i, w_i, 1 \leq i \leq n$ be the vertices of K_3 which are attached to the vertices of P_n. Let G $P_n \odot K_3$. The graph is displayed below:

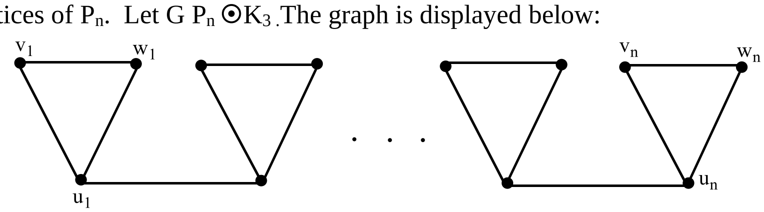

Figure .15

Here $p + q = 7n - 1$.

Define a function $f : V(G) \to \{1, 2, \ldots, p + q\}$ by

$$f(u_i) = 7i - 6,\ 1 \le i \le n,$$
$$f(v_i) = 7i - 4,\ 1 \le i \le n,$$
$$f(w_i) = \begin{cases} 7i - 3, & i = 1 \text{ and } n \\ 7i, & 2 \le i \le n-1 \end{cases}.$$

The edges are labeled with

$$f^*(u_1u_2) = 7,$$
$$f^*(u_iu_{i+1}) = 7i - 2,\ 2 \le i \le n-1,$$
$$f^*(u_iv_i) = 7i - 5,\ 1 \le i \le n,$$
$$f^*(u_iw_i) = 7i - 3,\ 1 \le i \le n,$$
$$f^*(v_iw_i) = \begin{cases} 7i - 2, & i = 1 \text{ and } n \\ 7i - 1, & 2 \le i \le n-1 \end{cases}.$$

Thus, the edge labels are distinct.

Hence $f(V(G)) \cup \{f(e) : e \in E(G)\} = \{1, 2, \ldots, p + q\}$.

Hence $P_n \odot K_3$ is a super cube root cube mean graph.

Example 2.18. Super cube root mean labeling of $P_5 \odot K_3$ is displayed below:

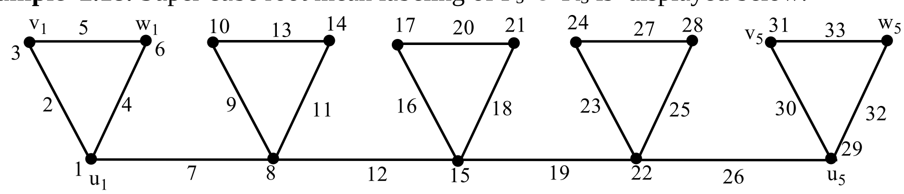

Figure : 16

Theorem 2.19. Fish graph is a super cube root cube mean graph.

Proof . Let G be a fish graph. Let $\{u_1, u_2, v_1, v_2, w_1, w_2\}$ be the vertices of G. It has n – vertices and n +1 edges. The graph is displayed below :

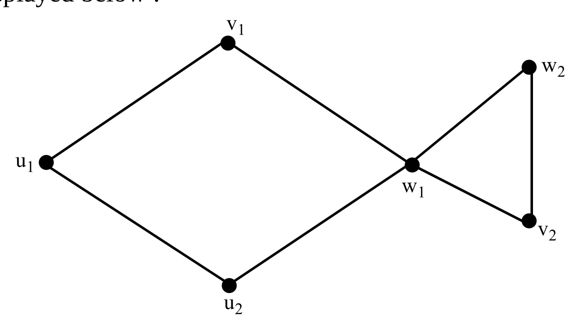

Figure : 17

Define a function $f : V(G) \to \{1, 2, \ldots, p + q\}$ by

$$f^*(u_i) = 2i - 1,\ 1 \le i \le 2,$$
$$f^*(v_i) = 5i,\ 1 \le i \le 2,$$
$$f^*(w_i) = 5i + 3,\ 1 \le i \le 2.$$

Thus , we get distinict edge labels. Hence, fish graph is a super cube root cube mean graph.

Example 2.20. Super cube root cube mean labeling of fish grapoh is shown below

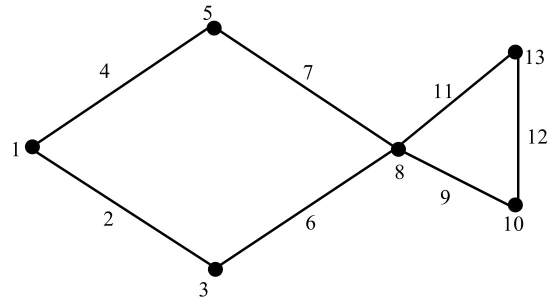

Figure : 18

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