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Structure of symmetric tensor of type (0, 2) on the complex tangent Bundle

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Abstract

In the present paper structures of symmetric of tensors of the type (0, 2) on the complex tangent bundle TM_{2n} of a complex manifold M_{2n} has been studied.

1. Introduction

Let M be a smooth manifold and TM [1] its tangent bundle also, let M_{2n} be a complex manifold [2] and TM_{2n} its tangent bundle [3]. Wong and Mock [4] have introduced the concept of M-tensor and M-connection on TM. In the paper [5] they have used these concepts to study the symmetric tensors of the type (0, 2) and tensor of type (1, 1) on TM. The main of the present paper is to study the symmetric tensors of the type (0,2) on the complex tangent bundle TM_{2n} .

Preliminaries

Let us fix our indices and notations Latin indices A,B,C,……,H,I,J,…..run over the range $\{1,2,\ldots,2n, \bar{1},\bar{2},\ldots\ldots,\bar{2n}\}$ and a, b, c,…….,h, I, j,….. the range $\{1,2,\ldots,2n, \bar{1},\bar{2},\ldots\ldots,\bar{2n}\}$ the Greek indices $\alpha,\beta,\gamma,\ldots$. $\tilde{n}=n+\alpha$ and $\bar{\tilde{n}}=\bar{n}+\bar{\alpha}$ .Summation over repeated indices ins always implied when matrices are used we denote there elements by Z^{α}, $A_{\alpha\beta}$ or p^{α}_{β} . In each case α denotes the row and β column of matrix A whose elements are $A_{\alpha\beta}$ will be denoted by $[A_{\alpha\beta}]$

Definition 1: An M_{2n} connections on TM_{2n} is the geometric objects determined by an assignment to each induced chart $(TU,(z,v))$ of an $2n\times 2n$ matrix function

$\Gamma=\left[\Gamma^{i}_{j}\right]=\left[\Gamma^{\lambda}_{\mu},\Gamma^{\bar{\lambda}}_{\mu},\Gamma^{\lambda}_{\bar{\mu}},\Gamma^{\bar{\lambda}}_{\bar{\mu}}\right]\overset{Def}{=}\left[\Gamma_1,\Gamma_2,\Gamma_3,\Gamma_4\right]$ such that

$$\Gamma'_1P_1=P_1\Gamma_1-\nabla_1P_1$$
$$\Gamma'_2P_2=P_2\Gamma_2-\nabla_2P_2$$
$$\Gamma'_3P_3=P_3\Gamma_3-\nabla_3P_3$$
[3.5] $$\Gamma'_4P_4=P_4\Gamma_4-\nabla_4P_4$$

Here Γ^{i}_{j} are components of the M_{2n} connections. (Γ_1,Γ_4) and (Γ_2,Γ_3) are pure and hybrid parts of the M_{2n} connections respectively.

Definition2:

A pure M_{2n}-tensor of type (r, s) on the complex tangent bundle TM_{2n} is the geometric object determined by an assignment to each induced chart $(TU,(z,v))$ of a set of $2n^{r+s}$ functions $S^{i_1\ldots\ldots i_r}_{j_1\ldots\ldots j_s}(z,v)$ that behave like the components of the tensor of type (r, s) on M_{2n} [3,5] i.e.

$$S^{\bar{i}_1\ldots\ldots\bar{i}_r}_{\bar{i}_1\ldots\ldots\bar{i}_s}$$

$$S^{i_1'\ldots\ldots i_r'}_{j_1'\ldots\ldots j_{s'}}=S^{i_1\ldots\ldots i_r}_{j_1\ldots\ldots j_s}\;S^{i_1\ldots\ldots i_r}_{j_1\ldots\ldots j_s}\;p^{i_1\ldots\ldots i_r}_{j_1'\ldots\ldots j_{s'}}+p^{\bar{i}_i\ldots\ldots\bar{i}_r}_{\bar{i}_1'\ldots\ldots\bar{i}_{s'}}\;S^{\bar{i}_1\ldots\ldots\bar{i}_r}_{\bar{i}_1\ldots\ldots\bar{i}_s}\;p^{\bar{i}_i\ldots\ldots\bar{i}_r}_{\bar{i}_1'\ldots\ldots\bar{i}_{s'}}\text{ on }TU\cap TU'$$

2. Symmetric Tensors of the type (0, 2) on M_{2n}

Let G be a non-zero symmetric tensor of the type (0, 2) on the complex tangent bundle TM_{2n} of a complex manifold M_{2n} which at present is not assumed to be of full rank or even of the same rank everywhere on TM_{2n} and let Γ be an M_{2n}-connection. Then we can express G on TU in pure and hybrid parts as

$$G=a_{ij}\omega^{i}\otimes\omega^{j}+a_{\bar{i}j}\omega^{\bar{i}}\otimes\omega^{j}+a_{i\bar{j}}\omega^{i}\otimes\omega^{\bar{j}}+a_{\bar{i}\bar{i}}\omega^{\bar{i}}\otimes\omega^{\bar{j}}$$

$$+h_{ji}\omega^{i}\otimes\omega^{\tilde{j}}\otimes h_{\bar{j}i}\omega\otimes\omega\,\bar{\tilde{j}}+h_{j\bar{i}}\omega^{\bar{i}}\otimes\omega^{\tilde{j}}+h_{\bar{j}\bar{i}}\omega^{\bar{i}}\otimes\omega^{\bar{\tilde{j}}}$$

$$+h_{ij}\omega^{\tilde{i}}\otimes\omega^{j}+h_{\bar{i}i}\omega^{\bar{\tilde{i}}}\otimes\omega^{j}+h_{i\bar{}}\omega^{\tilde{i}}\otimes\omega^{\bar{j}}+h_{\bar{\bar{i}}}\omega^{\vec{\bar{\tilde{\tilde{i}}}}}\otimes\omega^{\vec{j}}$$

$$+c_{ij}\omega^{\tilde{i}}\otimes\omega^{\tilde{i}}+c_{\bar{ij}}\omega^{\bar{\tilde{i}}}\otimes\omega^{\tilde{j}}+c_{i\bar{j}}\omega^{\tilde{i}}\otimes\omega^{\tilde{j}}+h_{\bar{\bar{i}}}\omega^{\vec{\bar{\tilde{\tilde{i}}}}}\otimes\omega^{\bar{\tilde{\tilde{j}}}}$$

Where $\left[\omega^{\alpha}\right]$ is the co-frame on TU adopted to Γ and $a=\left[a_{ij}\right]$, $h=\left[h_{ij}\right]$ and $c=\left[c_{ij}\right]$ are M_{2n}-tensors of the type (0,2) on TM_{2n}.

Relative to the M_{2n}-connection Γ, the frame components matrix of G is

$$\begin{bmatrix} a & a^{*} & h^{t} & h^{*t} \\ *a & \bar{a} & *h^{t} & \overline{h^{t}} \\ h & h* & c & c* \\ *h & \bar{h} & *c & \bar{c} \end{bmatrix}$$

Theorem2.1: The most general symmetric tensor G of type (0, 2) on the complex bundle TM_{2n} has matrix of the form

$$G=\begin{bmatrix} a+\Gamma^{t}c\Gamma+h^{t}\Gamma+\Gamma^{t}h & a^{*}+\Gamma^{*t}c^{*}\Gamma^{*}+h^{*t}\Gamma^{*}+\Gamma^{*t}h^{*} & \Gamma^{t}c+h^{t} & \Gamma^{*t}c^{*}+h^{*t} \\ {}^{*}a+{}^{*}\Gamma^{t}\,{}^{*}c\,{}^{*}\Gamma+{}^{*}h^{t}\Gamma+{}^{*}\Gamma^{t}\,{}^{*}h & \bar{a}+\bar{\Gamma}^{t}\bar{c}\bar{\Gamma}+\bar{h}^{t}\bar{\Gamma}+\bar{\Gamma}^{t}\bar{h} & {}^{*}\Gamma^{t}\,{}^{*}c+{}^{*}h^{t} & \bar{\Gamma}^{t}\bar{c}+\bar{h}^{t} \\ c\Gamma+h & c^{*}\Gamma^{*}+h^{*} & c & c^{*} \\ {}^{*}c\,{}^{*}\Gamma+{}^{*}h & \bar{c}\bar{\Gamma}+\bar{h} & {}^{*}c & \bar{c} \end{bmatrix}$$

[2.1]Where c is the M_{2n}-tensor associated with G, Γ is an M-connection and h are M_{2n}-tensor of the type (0, 2) on M_{2n}.

Proof: We know that the adopted frame and the natural frame are related by

$$\left[\omega^{\alpha}\right]=N\left[dz^{\alpha}\right],\left[d_{\beta}\right]=\left[\partial/\partial z^{\beta}\right]N^{-1}$$

Where

$$N=\begin{bmatrix} I & 0 \\ \Gamma & I \end{bmatrix}, N^{-1}=\begin{bmatrix} I & 0 \\ -\Gamma & I \end{bmatrix}$$

Relative to M_{2n}-connection, the frame components matrix of G is

$$\begin{bmatrix} a & a^* & h^t & h^{*t} \\ {}^*a & a & {}^*h^t & \bar{h}^t \\ h & h^* & c & c^* \\ {}^*h & \bar{h} & {}^*c & \bar{c} \end{bmatrix}$$

The tensor G can also be uniquely expressed as the following sum of three symmetric tensors of the type (0, 2) on TM_{2n}.

$$\begin{bmatrix} a-c & a^*-c^* & 0 & 0 \\ {}^*a-{}^*c & \bar{a}-\bar{c} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} c+\Gamma^t c\Gamma & c^*+\Gamma^{*t}c^*\Gamma^* & \Gamma^t c & \Gamma^{*t}c^* \\ {}^*c+{}^*\Gamma^t\,{}^*c\,{}^*\Gamma & \bar{c}+\bar{\Gamma}^t\bar{c}\bar{\Gamma} & {}^*\Gamma^t\,{}^*c & \bar{\Gamma}^t\bar{c} \\ c\Gamma & c^*\Gamma^* & c & c^* \\ {}^*c\,{}^*\Gamma & \bar{c}\bar{\Gamma} & {}^*c & \bar{c} \end{bmatrix}$$

$$+\begin{bmatrix} h^t\Gamma+\Gamma^t h & h^{*t}\Gamma^*+\Gamma^{*t}h^* & h^t+ & h^{*t} \\ {}^*h^{t^*}\Gamma+{}^*\Gamma^t\,{}^*h & \bar{h}^t\bar{\Gamma}+\bar{\Gamma}^t\bar{h} & {}^*h^t & h^t \\ h & h^* & 0 & 0 \\ {}^*h & \bar{h} & 0 & 0 \end{bmatrix}$$

Let $\tilde{\Gamma}$ be another M_{2n} -connection and

$$\begin{bmatrix} \tilde{a} & \tilde{a}^* & \tilde{h}^t & \tilde{h}^{*t} \\ {}^*\tilde{a} & \bar{\tilde{a}} & {}^*\tilde{h}^t & \bar{\tilde{h}}^t \\ \tilde{h} & \tilde{h}^* & \tilde{c} & \tilde{c} \\ {}^*\tilde{h} & \tilde{h} & {}^*\tilde{c} & \bar{\tilde{c}} \end{bmatrix}$$

The frame components of G relative to $\tilde{\Gamma}$. Then we have

(2.2) $\tilde{c}=c, \bar{\tilde{c}}=\bar{c}$

(2.3) $\tilde{h}=h-cT, \bar{\tilde{h}}=\bar{h}-\bar{c}\bar{T}$ and $\tilde{a}=a-T^t\,\mathrm{c}T-\bar{\tilde{h}}^t\,T-T^t h$,

where $T=\tilde{\Gamma}-\Gamma, \bar{T}=\bar{\tilde{\Gamma}}-\bar{\Gamma}$

These relations suggest that we can express a symmetric tensor of the type (0,2) on TM_{2n} as a certain equivalence class. In fact let $(c, \bar{c})$ be a symmetric M_{2n} tensor of type (0,2) on M_{2n}.

Theorem 2.2: The most general non-singular symmetric tensor of the type (0,2)on the complex tangent bundle TM_{2n} has a component matrix of the form

$$\begin{bmatrix} a+\Gamma^t c\Gamma & a^*+\Gamma^{*t}c^*\Gamma^* & \Gamma^t c & \Gamma^{*t}c^* \\ {}^*a+{}^*\Gamma^t\,{}^*c\,{}^*\Gamma & \bar{a}+\bar{\Gamma}^t\bar{c}\bar{\Gamma} & {}^*\Gamma^t\,{}^*c & \bar{\Gamma}^t\bar{c} \\ c\Gamma & c^*\Gamma^* & c & c^* \\ {}^*c\,{}^*\Gamma & \bar{c}\bar{\Gamma} & {}^*c & \bar{c} \end{bmatrix}$$

Where Γ as an M_{2n}-connection, a is a symmetric M_{2n}-tensor and c is associated M_{2n}-tensor on M_{2n}.

Proof: Suppose that G is a symmetric tensor of type (0,2) satisfying the condition of equivalence. Then there is a unique M_{2n} connection Γ such that $H_\Gamma = V^1$.Let $[D_\beta]$ is the frame adopted to Γ, then $G(D_i, D_j) = 0$. It follows that the frame component matrix of G is of the form $\begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$, where $c = A(\mathrm{G})$ and $\mathrm{a} = [a_{ij}]$ is the symmetric M_{2n}-tensor determined by $a_{ij} = \mathrm{G}(D_i, D_j)$ and is therefore the restriction of G to $H_\Gamma = V^1$.In terms of c, Γ and a, we have the theorem.

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