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Spread of COVID-19 in India: A Mathematical Model

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Abstract: *The COVID-19 pandemic has been the greatest threat to human lives of the entire world since January 2020. In the present article, we discuss a mathematical model regarding the spread of COVID-19 in India. This model is aimed at finding the nature of time dependence of the number of symptomatic patients, officially recorded in the country, during the period from 01 March 2020 to 23 April 2020. The number of persons infected with the coronavirus disease, as declared by the government on a regular basis, is most probably the number of patients who have experienced the symptoms of the disease. The present study is based on a differential equation that has been formed here to find how the number of asymptomatic patients increases with time. The number of symptomatic patients has been estimated from its solution. The nature of its time evolution is found to be quite consistent with the data obtained from government records, for a certain set of parameter values of the model. Using this particular set, we have discussed the impact of imposition of a countrywide lockdown and its withdrawal.*

Keywords: *Novel Coronavirus(SARS-CoV-2), COVID-19, Social Distancing, Lockdown, Spread of an Epidemic, Mathematical Model.*

I. Introduction

A new virus, called SARS-CoV-2, is responsible for an outbreak of respiratory illness known as COVID-19, which has been found to spread globally from People's Republic of China since December 2019 [1, 2]. SARS is an acronym for Severe Acute Respiratory Syndrome. Since the RNA genome of the virus is about 82% identical to the SARS coronavirus (SARS-CoV), this was named SARS-CoV-2, with both viruses belonging to clade *b* of the genus Beta coronavirus [1-3]. The outbreak of novel coronavirus disease (COVID-19) was declared a pandemic by the World Health Organization (WHO) on 11 March 2020 [4].They had earlier declared this outbreak a Public Health Emergency of International Concern on 30 January 2020 [4]. As of 26 April 2020, a total of 2,917,073 confirmed cases of novel coronavirus disease and 203,545 deaths have been reported globally [5]. This disease was first detected in Wuhan (Hubei, China), as a local outbreak of pneumonia of some unknown cause in December 2019 [1, 2, 6, 7]. Within the first two months of the outbreak, the epidemic spread around the world with an alarming rapidity. As of March 08 2020, a total of 80,868 confirmed cases and 3,101 deaths had been reported in Chinese mainland by National Health Commission of China, and 90 other countries were affected [8]. Along with the humans, other animal species are vulnerable to such coronavirus infections, which cause a variety of severe diseases, including Severe Acute Respiratory Syndrome (SARS) and Middle East Respiratory Syndrome (MERS) [9, 10]. The genome of COVID-19 virus consists of nearly 30,000 nucleotides, and its replicase gene encodes two overlapping polyproteins, pp1a and pp1ab, which are required for viral replication and transcription [2, 11]. As of 26 April 2020, there was no vaccine and no antiviral medicines against COVID-19 [4].

The first case of coronavirus pandemic in India was reported on 30 January 2020. As of 26 April 2020, a total of 26,917 cases of infection, 5,914 recoveries and 826 deaths in the country have been confirmed by the Ministry of Health and Family Welfare [12]. The central and the state governments have taken several measures to spread awareness regarding COVID-19 and also to implement social distancing of its citizens to break the chain of transmission of the disease. On 24 March 2020, a nationwide lockdown was imposed for 21 days. On 14 April

2020, this lockdown was extended till 03 May 2020. P. Chatterjee et al. have conducted a detailed study to gather evidence that can guide research activities towards the prevention and control such an epidemic in India [13]. Apart from building databases, some mathematical models have been constructed, which can help the policy makers to make proper plans to prevent the spread of this disease [14, 15].

In the present study we have formulated a simple mathematical model to account for the rise in the number of COVID-19 cases in India. The official figures regarding these cases hardly include the asymptomatic ones, mainly due to the difficulty of carrying out tests in sufficient numbers. It has been assumed for the present formulation that, once a symptomatic case is detected, the patient is immediately put into isolation, preventing him completely from playing any role in the transmission of the disease. We have constructed a model to find the time dependence of the number of asymptomatic cases, from which we have estimated the number of symptomatic cases. For a certain set of parameter values of the model, our findings are found to agree sufficiently with the data obtained from the government sources. For this purpose we have used the COVID-19 data, for the period from 01 March 2020 to 23 April 2020, obtained from the websites of the Ministry of Health and Family Welfare, Govt. of India and the Indian Council of Medical Research (ICMR) [12, 16]. This model has shown the impact of imposition and withdrawal of lockdown. It clearly shows that a proper lockdown reduces the rate of transmission of the disease sufficiently. All these findings have been shown graphically.

II. A Dynamical Model

We propose the following differential equation to find the time evolution of the number of asymptomatic cases of COVID-19 in the country.

$$\frac{dy}{dt} = f_1(y) - f_2(y) \tag{1}$$

In equation (1), the variable y denotes the total number of asymptomatic carriers at any instant of time *t*. The function $f_1(y)$ denotes the number of persons getting infected per unit time. They are likely to remain asymptomatic over the incubation period of the virus. The function $f_2(y)$ stands for the number of persons who are diagnosed as symptomatic and get isolated per unit time. Here it is assumed that no new asymptomatic carrier is allowed to enter the region under consideration.

The number of persons getting infected per unit time, through contacts with asymptomatic carriers, must depend upon the pattern of social mixing of these carriers. One can always think of (or estimate) an average number of people getting infected by each such carrier per unit time. If *n* be the number of persons coming in contact with each carrier per unit time, and β be the fraction of them getting this infection, we can write,

$$f_1(y) = \beta n y \tag{2}$$

Here, β is actually the probability that a person, in proximity of an asymptomatic carrier, would be infected by the virus. Its value certainly depends upon the average awareness level and cautiousness of the people.
We have assumed an allometric relation to exist between the function $f_2(y)$ and *y*. It can be expressed as,

$$f_2(y) = \lambda y^k \tag{3}$$

Here k is the allometric exponent and λ is the constant of proportionality. Using equations (2) and (3) in equation (1), one gets,

$$\frac{dy}{dt} = \beta n y - \lambda y^k \tag{4}$$

The solution of equation (4) is given by,

$$y = \left[\frac{\lambda}{\beta n} + \frac{1}{\beta n} Exp\{\beta n(1-k)(t+c)\}\right]^{\frac{1}{1-k}} \tag{5}$$

Here c is the constant of integration. Taking $y = y_0$ at $t = t_0$, the value of c is obtained as,

$$c = \frac{\ln\{\beta n({y_0}^{1-k} - \lambda/\beta n)\}}{\beta n(1-k)} - t_0 \tag{6}$$

Substituting for *c* in equation (5), from equation (6), we get,

$$y = \left[\frac{\lambda + (\beta n {y_0}^{1-k} - \lambda) Exp\{\beta n(1-k)(t-t_0)\}}{\beta n}\right]^{\frac{1}{1-k}} \tag{7}$$

Equation (7) shows how the number of asymptomatic patients increases with time. This function can be used in equation (3) to determine the time evolution of the number of symptomatic cases. The number of symptomatic patients, registered in a country like India, is very often shown as the total number of persons infected with COVID-19, due to the lack of information regarding the number of asymptomatic patients in most cases. The number of cases of infection, registered upto a certain day, can thus be predicted by taking the sum of the values of the function $f_2(y)$ for all these days from the beginning. The unit of time, in this case, is taken to be a single day.

It has been observed that, to prevent the spread of the virus, lockdown has been imposed in almost all countries to maintain as much social distancing as possible. Social distancing is supposed to reduce the value of the parameter *n*, which denotes the number of persons coming in direct contact with each asymptomatic patient per unit time. This model allows us to examine the effect of social distancing by finding how y changes with time for a smaller value of *n*.

Let us suppose that a complete lockdown has been imposed, after a certain number of days (say d_1) have passed following the detection of the disease in the country. Let d_2 be the number of days for which the lockdown is continued. The time evolution of the number of asymptomatic carriers (denoted by Y here), in that case, would be expressed as,

$$Y = g_1(t) y_{d_1} + g_2(t) y_{d_2} \tag{8}$$

Here, y_{d_1} and y_{d_2} are the functions that determine, respectively, the time variation of Y for the initial d_1 number of days (prior to lockdown) and the d_2 number of days (during lockdown). Each of them has an expression that is identical to the expression of *y*, given by equation (7). The roles of the functions, $g_1(t)$ and $g_2(t)$, are to ensure that $Y = y_{d_1}$ before lockdown and $Y = y_{d_2}$ during lockdown. Letting n_1 and n_2 to be the values of *n*, for the pre-lockdown and lockdown periods respectively, one must have $n_1 > n_2$, as a consequence of lockdown which is likely to enhance the social distancing. The value of the parameter *n* is greater for y_{d_1} than for y_{d_2}. The values of the parameters t_0 and y_0, for the function y_{d_2}, should be such that $y_{d_1} = y_{d_2}$ just at the beginning of the lockdown period, for the sake of continuity. The function $g_1(t)$ is chosen to be such that $g_1(t) = 1$ for $0 \le t \le d_1$ and $g_1(t) = 0$ for $t > d_1$. The function $g_2(t)$ is such that $g_2(t) = 0$ for $t \le d_1$ and $g_2(t) = 1$ for $d_1 < t \le d_2$. Keeping in mind all these requirements, we have chosen the following functional forms for $g_1(t)$ and $g_2(t)$.

$$g_1(t) = \frac{(1 + \tanh \gamma t)[1 + \tanh \gamma (d_1 - t)]}{4} \tag{9}$$

$$g_2(t) = \frac{\{1 + \tanh \gamma (t - d_1)\}[1 + \tanh \gamma (d_1 + d_2 - t)]}{4} \tag{10}$$

Equation (9) approximates a rectangular pulse of unit height, having a width (or duration) of d_1, over the span of $0 \le t \le d_1$. Equation (10) approximates a rectangular pulse of unit height, with a width (or duration) of d_2, over the span of $d_1 \le t \le d_2$. In each of these two functions, one must choose a large value for the parameter γ (in comparison to pulse widths), to make sure that its shape is as close to a rectangle as possible.
The number of symptomatic cases, registered per unit time, can be obtained from equation (8), with the help of equation (3). The functional forms for $g_1(t)$ and $g_2(t)$ are such that, when any of them is unity, the other is zero. Taking a single day to be the unit of time, the number (N_i) of symptomatic cases detected on the i^{th} day would be,

$$N_i = \lambda Y^k = \lambda \left[g_1(t) y_{d_1} + g_2(t) y_{d_2}\right]^k \tag{11}$$

While using the above expression of N_i, one must replace the variable *t* by the variable *i* (counted in days), in the expressions of $g_1(t)$, $g_2(t)$, $y_{d_1}$and y_{d_2}, as we have shown here from equation (12) onwards.
Using equation (11), the total number (S_L) of symptomatic cases recorded till a certain day (say j^{th} day) is given by,

$$S_L(j) = \sum_{i=1}^{j} \lambda\left[g_1(i)y_{d_1}(i) + g_2(i)y_{d_2}(i)\right]^k \quad (12)$$

Here, the subscript *L* in the above expression corresponds to the situation created by the imposition of lockdown. Without any lockdown being imposed, this number would be,

$$S_{WL}(j) = \sum_{i=1}^{j} \lambda y(i)^k \quad (13)$$

Here, the subscript *WL* in the above expression corresponds to the state without any lockdown having been imposed. In equation (13), $y(i)$ is the same as *y* in equation (7), with *t* being replaced by *i* in its expression.
To estimate the effect of lockdown theoretically, we define a ratio of the number of symptomatic cases recorded till a certain day beyond d_1 to the number of such cases found upto the same day without any lockdown being imposed. This ratio (R_m), for the $(d_1 + m)^{th}$ day, is given by,

$$R_m = \frac{S_L(d_1+m)}{S_{WL}(d_1+m)} = \frac{\sum_{i=1}^{d_1+m} \lambda\left[g_1(i)y_{d_1}(i)+g_2(i)y_{d_2}(i)\right]^k}{\sum_{i=1}^{d_1+m} \lambda y(i)^k} \quad (14)$$

The greater the effect of lockdown, the smaller would be the ratio R_m. This ratio is supposed to be a fraction whose value should decrease as *m* increases.
Let us suppose that the lockdown is withdrawn for a period of d_3 number of days (beyond d_2) and again imposed for a period of d_4 number of days. Following the logic behind the formulation of equation (8), the time variation of the number of asymptomatic patients, in this case, would be expressed as,

$$Y = g_1(t)y_{d_1} + g_2(t)y_{d_2} + g_3(t)y_{d_3} + g_4(t)y_{d_4} \quad (15)$$

Here $g_3(t)$ and $g_4(t)$ are given by,

$$g_3(t) = \frac{\{1+\tanh\gamma(t-d_1-d_2)\}[1+\tanh\gamma(d_1+d_2+d_3-t)]}{4} \quad (16)$$

$$g_4(t) = \frac{\{1+\tanh\gamma(t-d_1-d_2-d_3)\}[1+\tanh\gamma(d_1+d_2+d_3+d_4-t)]}{4} \quad (17)$$

Here, $g_3(t) = 1$ for $d_1 + d_2 \leq t \leq d_1 + d_2 + d_3$ and zero otherwise.
and $g_4(t) = 1$ for $d_1 + d_2 + d_3 \leq t \leq d_1 + d_2 + d_3 + d_4$ and zero otherwise. They represent two rectangular pulses of widths d_3 and d_4 respectively and of unit height (for a sufficiently high value of γ).
Each of the functions, denoted by the symbol y_d in equation (15), is identical in form to the expression of *y*, given by equation (7). Each of them has a unique set of values for the parameters *n*, t_0 and y_0. For continuity at the interface of the two spans, d_2 and d_3, we must have $y_{d_2} = y_{d_3}$, and similarly, at the interface of the two spans, d_3 and d_4, we must have $y_{d_3} = y_{d_4}$. To ensure this, the values of t_0 and y_0 in their expressions should be calculated accordingly, in accordance with the discussion following equation (8). The value of *n* is less for the lockdown period than that for the normal period.
Following the method, discussed previously for the formulation of equation (11), the number (N_i) of symptomatic cases detected on the i^{th} day would be,

$$N_i = \lambda\left[g_1(i)y_{d_1}(i) + g_2(i)y_{d_2}(i) + g_3(i)y_{d_3}(i) + g_4(i)y_{d_4}(i)\right]^k \quad (18)$$

Using equation (18), the total number (S_L) of symptomatic cases recorded upto a certain day (say j^{th} day) is given by,

$$S_L(j) = \sum_{i=1}^{j} \lambda\left[g_1(i)y_{d_1}(i) + g_2(i)y_{d_2}(i) + g_3(i)y_{d_3}(i) + g_4(i)y_{d_4}(i)\right]^k \quad (19)$$

If the lockdown is imposed and withdrawn alternately, for N number of times, a generalized expression for $S_L(j)$ would be,

$$S_L(j) = \lambda \sum_{i=1}^{j} \left[\sum_{s=1}^{N} g_s(i) y_{d_s}(i)\right]^k \tag{20}$$

By fitting $S_L(j)$ (of eqn. 12) numerically to the registered data of symptomatic patients (Figure 1), we have found $k = 0.283, \beta = 0.1, \lambda = 0.027, y_0 = 1000\ at\ t_0 = 1$ with n being 10.2 and 3 for the functions $y_{d_1}(i)$ and $y_{d_2}(i)$, respectively.

III. Graphical Interpretation

Figure 1 shows the time evolution of the number of symptomatic patients, through a plot of data obtained from the government records and also the data generated by the present mathematical model. The circles denote the numbers registered in India from 01 March 2020 (i.e. j = 1) to 23 April 2020 (i.e. j = 54). The continuous curve represents the values predicted by the present model. The actual observations are found to be in good agreement with the theoretical predictions. The parameter values for which this agreement has been possible are: $n_1 = 10.2$, $k = 0.283$, $\beta = 0.1$, $\lambda = 0.027$, $n_2 = 3$, $t_0 = 1$ and $y_0 = 1000$. Here, $S(j)$ is the same as $S_L(j)$ of equation (12).

Figure 2 shows the effect of lockdown on the time variation of the number of symptomatic patients. The red and black curves represent the situations, with and without the imposition of lockdown, respectively, as predicted by the present model. Here, j = 1 corresponds to 01 March 2020. This plot shows that the number would have been almost two orders of magnitude higher, on the 54^{th} day since 01 March 2020, without the imposition of lockdown.

Figure 3 shows a plot of R versus m where R denotes the ratio of the number of symptomatic patients under lockdown to the number without lockdown (from the present model), on the m^{th} day following the imposition of lockdown (i.e. from $m = 1$which is the first day of the lockdown period). It shows that as n_2 decreases, R decreases. Smaller values of n_2 indicate more stringent implementation of social distancing during lockdown.

Figure 4 shows the time evolution of the number of symptomatic patients, for three phases: (i) before, (ii) during and (iii) beyond lockdown, assuming the lockdown to persist for 40 days (25 March to 03 May, 2020), as per the last announcement, made on 14 April 2020. Here, j = 1 on 01 March 2020. It is evident from the plot that the rate of rise in the number of cases is obviously smaller during the lockdown period, in comparison to other periods.

IV. Conclusions

The predictions, regarding the number of COVID-19 cases in India, based on the present study, are found to be close to the values recorded by government. This model shows that the imposition of a countrywide lockdown plays a very important role in restraining the spread of the disease. By increasing the degree of social distancing, one can reduce the value of the parameter n, which would significantly reduce the speed at which the disease is transmitted. By increasing the social awareness level regarding the precautions to be taken by each individual, one can reduce the value of the parameter β in equation (4). Through a proper awareness campaign and a strict implementation of lockdown, the value of the product βn can be so small that $\frac{dy}{dt}$ is negative (in eqn. 4).The number of asymptomatic carriers of COVID-19 would then be decreasing with time, causing a fall in the number of symptomatic patients recorded every day. In the present article, we have not shown any estimate regarding the number of asymptomatic patients, obtained from this model, because that can't be verified by the available data.

This model can be improved by choosing the functional forms of $f_1(y)$ and $f_2(y)$ on the basis of evidence gathered in the form of data collected over a long span of time from different parts of the country. One may also consider the time dependence of the parameters β and n to have a more realistic estimate of y. This model can be mathematically modified by taking into consideration the immense diversity of plans regarding disaster management in different parts of such a large country. This model has an underlying assumption that, once a symptomatic case is detected, the patient is immediately put into complete isolation. In many cases this is far from what happens in reality. There are also situations where we have a delayed diagnosis of a symptomatic case. All these factors have to be taken into consideration to construct a better mathematical model.

FIGURES

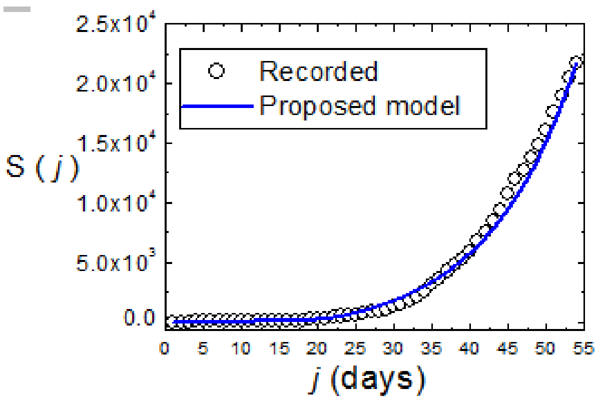

Figure 1: Variation of the number of symptomatic patients as a function of time. The circles denote the numbers registered in India during the period from 01 March 2020 (i.e. $j = 1$) to 23 April 2020 (i.e. $j = 54$). The continuous curve represents the values predicted by the present model.

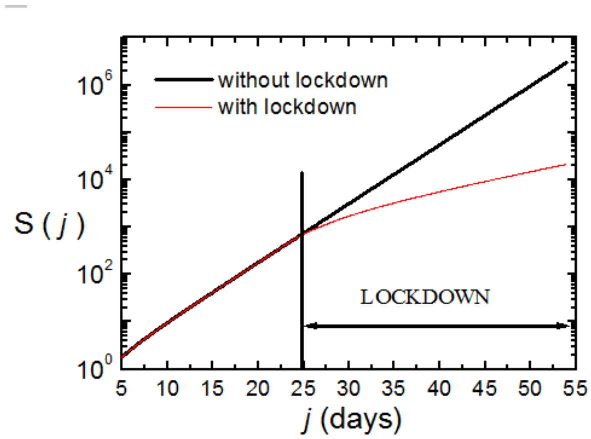

Figure 2: Variation of the number of symptomatic patients as a function of time, plotted in log scale. The red and black curves correspond to situations, with and without lockdown, respectively, as predicted by the present model. Here, $j = 1$ on 01 March 2020 and $j = 54$ on 23 April 2020.

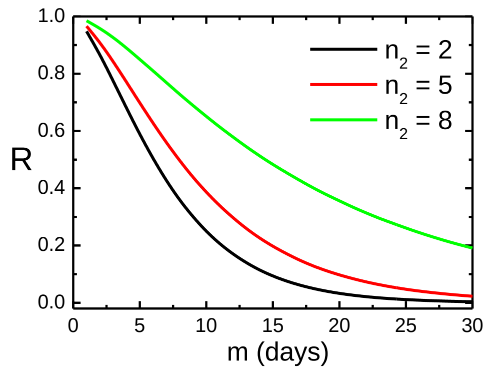

Figure 3: A plot of *R* versus *m* where *R* is the ratio of the number of symptomatic patients under lockdown to the number without lockdown (obtained from the present model), on the m^{th} day following the imposition of lockdown. Smaller values of n_2 indicate greater degree of social distancing.

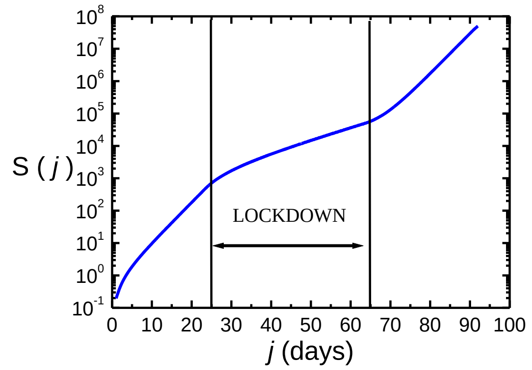

Figure 4: Variation of the number of symptomatic patients as a function of time for three phases: (i) before, (ii) during and (iii) beyond lockdown, assuming the lockdown to continue for 40 days (25 March to 03 May, 2020). Here, $j = 1$ on 01 March 2020 and $j = 54$ on 23 April 2020.

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