Journal of Science and Technology
ISSN: 2456-5660 Volume 8, Issue 2 (February -2023)
www.jst.org.in *DOI:https://doi.org/10.46243/jst.2023.v8.i02.pp18- 25*

Designing Reliable Trans Multiplexers in Noisy Environments

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To Cite this Article

Y SIVA PRASAD , P SRINIVASULU, "Designing Reliable Trans Multiplexers in Noisy Environments''*Journal of Science and Technology, Vol. 08, Issue 07,- NOV 2022, pp18-25*

Article Info

Received: 21-1-2023 Revised: 01-02-2023 Accepted: 10-02-2023 Published: 24-02-2023

Abstract

An optimum strategy for the design of trans multiplexer systems in the presence of noise is introduced. We consider the challenge of reducing the reconstruction error for a trans multiplexer system with fixed transmit terms and a target attenuation of crosstalk between bands. We may make the suboptimal reconstruction error system less vulnerable to variations in input signals and channel disturbances by using mixed H2/H optimization. Crosstalk between channels is to be anticipated due to the overlap of neighbouring subchannels. Crosstalk attenuation is posed as a H optimization problem and is decomposed into a set of linear matrix inequalities (LMIs). The simulation results show that the suggested design outperforms the state-of-the-art designs.

INTRODUCTION

Originally intended to transfer data between time division multiplexed (TDM) format and frequency-division multi flexed (FDM) format, trans multiplexers (TMUX) were researched by Bellanger and Daguet [1] in the early 1970s for telephone applications. They have been put to good use in facilitating interactions amongst several participants. Transmission of multiple data signals over a single channel using the frequency-division multiplexing (FDM) technique is well suited to a multi-input multi-output (MIMO) M-band conventional TMUX system (Figure 1) with critical sampling (i.e., all interpolation factors equal to band number, also called as minimally interpolated TMUX in [2]). Transmitters (the left filter bank) Fi(z) in a conventional distortion-free (C(z) = 1 and r(n) = 0 in Figure 1) TMUX system typically span a variety of uniform frequency ranges. The signal q(n) is the sum of M adjacent frequency bands (passbands of the filters) containing the signals ui(n), i = 0, 1,..., M 1. Using ideal bandpass filters for the transmitters Fi(z), i = 0, 1, M 1, we can think of p(n) as an FDM version of the individual signals ui(n), and the receivers (the right filter bank) Hk(z) break this signal down into v i(n), i = 0, 1, M 1, with the decimated version of v i(n) being the reconstructed signal si(k).

Thus, the TMUX system may be seen as the dual system of the sub band filter bank system [3], converting from TDM and FDM and back to TDM. However, with the TMUX system, the adjacent spectra will tend to overlap if the transmitters Fi(z) are not optimal. Similarly, if the receivers Hi(z) are imperfect, the intended signal input is(k) and other input signals sly(k), l = I, will both contribute to the output signal is(k) in the itch band. Crosstalk [4] refers to the transfer of information across frequency bands. The down sampling procedures and the imperfection of the transmitting filters Fi(z) are the primary sources of the crosstalk phenomena that plagues TMUX systems. Numerous researches have been conducted in the past. Since there is no

overlap between signals in neighbouring bands in the FDM format, crosstalk may be removed intuitively by using nonoverlapped transmitters Fi(z) and band limiting the signals is(k) to || I with I . In other words, even if the filters contain a nonzero transition band [5, there exists a guard band between neighbouring frequency bins that prevents crosstalk between adjacent signals. For receivers Hi(z), a bigger guard band means a wider transition band may be used at reduced cost. The transmission method does not make full use of the channel capacity due to the presence of guard bands. If the filters at the transmitter, shown by the symbols Fi(z), have an extremely sharp cut-off and an equal bandwidth of /M, then the whole channel may be used. Good approximations of such perfect filters are costly, and ideal filters themselves are obviously unreal sizable. Even though perfect filters are impractical, crosstalk in TMUX systems may be eliminated using well-designed separation filters.

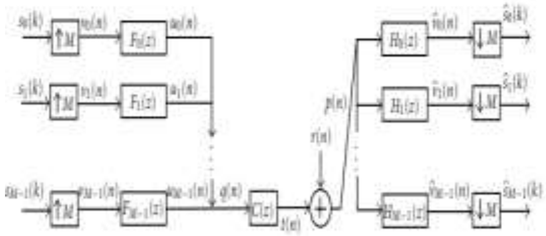

Figure 1: TMUX model with channel and channel noise.

example, Vetterli [6]. In this approach, crosstalk is permitted in TDM → FDM converter but is cancelled at the FDM → TDM stage. That is, even if there are no guard bands (thereby permitting crosstalk), we can eliminate the crosstalk in a manner analogous to aliasing cancellation in maximally decimated filter banks by a careful choice of transmitters and receivers. By this approach, the filters Hi(z) and Fi(z) are more economical than those in conventional designs. In fact, note that under certain condition perfect symbol recovery may be possible even with nonideal filters having overlapping responses, for instance, with the so-called biorthogonal filter bank [7]. For noise-free TMUX system, a lot of conventional researches have been devoted to exploit the perfect reconstruction property. As such, it has been studied from the point of view of periodically time-varying (PTV) filters in [8, 9], with the technique of the selection of PTV filters poles and zeros. In [10], an H2 optimization approach is used to design nonuniform-band TMUX systems, resulting in Near PR (NPR) TMUX systems. Moreover, since the quadrature mirror filter (QMF) bank and the TMUX system are dual to each other, the design of PR TMUX system can be solved by design PR QMF system, as discussed in [5]. Unfortunately, this perfect recovery is achieved under the assumption that channel effects including channel distortion and additive channel noises are negligible. For practical distorted channels, the orthogonality between bands is destroyed at the receiver, causing in most cases unacceptable performance degradation. A practical channel model is shown in Figure 1 which consists of linear FIR filter C(z), with order L< M

(a reasonable assumption after channel equalization), and with additive noise r(n), see [11]. The composite signal p(n) is a distorted and noisy version of

$$\{s_0(k), s_1(k), \ldots, s_{M-1}(k)\}.$$

For this practical noisy TMUX system, in [12], Wiener filtering approach is presented via the least-squares method to maintain the reconstruction performance, also, Chen et al. proposed a series of studies to deal with the signal reconstruction problem from the H2 optimal point of view [13–15], and recently, an MMSE approach is proposed for perfect DFT-based DMT system design [11], with the major shortcoming that the statistical properties of input and noises must be known. To improve it, H∞ optimization or minimax approach is developed in [16]. Moreover, in [17], a mixed H2/H∞ design is developed for TMUX system with additive noise, but with much conservatism due to adopting the same Lyapunov matrix for characterizing both the H2 and H∞ performances. In this study, we focus on a critically sampled TMUX system. It is assumed that all users are independent, that is, si is independent of sj for i = j; and each band is allowed to have different delays di for constructing its input. Both the transmitters and receivers are assumed to be FIR filters and channel noise r(n) is a white noise [11]. We address the problem of minimizing the reconstruction error while ensuring that the crosstalk is below certain level in the presence of channel noise. We will first design optimal and robust receivers to reconstruct the input signals with the optimal reconstruction error in the noisy channel. For the crosstalk optimization problem, some H∞ constraints are added to ensure the TMUX system within desired crosstalk attenuation levels. Our solution is given in terms of linear matrix inequalities (LMIs) which can be solved easily by convex optimization [18]. As illustrated later, compared with the existing TMUX design method via LMI technique [17], the proposed method embodies two obvious advantages. First, when the reconstruction performance is concerned, the proposed mixed H2/H∞ optimization method provides less conservative results. Second, a multiobjective TMUX system issue has been explored in this study, in particular, the issue on both optimal reconstruction performance and the crosstalk attenuation is novelty formulated and solved via LMI techunique.

$$\{s_0(k), s_1(k), \ldots, s_{M-1}(k)\}.$$

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H2 OPTIMIZATION ON RECONSTRUCTION ERROR

In this section, we will establish the state-space model of the noisy TMUX system first, then formulate its H2 optimization by LMIs. Remark 1. In a practical TMUX system, most TMUX systems apply an FIR equalizer in order to shorten the effective length of the physical channel impulse response, modeled as an FIR filter C(z) with order L (usually, the order L of C(z) is smaller than the interpolation factor M [2], which is called as the LS shortening [19]), and may be multichannel case Ci(z) (i = 0, 1, ... , M − 1) in some TMUX system applications. For the convenience of further discussion, throughout the paper, we will combine each transmitting filter Fi(z) with subchannel Ci(z) together, and describe the Ci(z)Fi(z) as new transmitting filter Fi(z), without specific explanation.

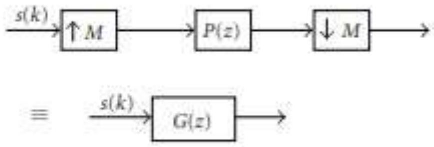

Figure 2: The polyphase identity

MIXED H2/H∞ OPTIMIZATION ON RECONSTRUCTION ERROR

It is well known that one of the major drawbacks of H2 optimization is that the statistical properties (or the models) of the input signals and channel noises must be well known beforehand. To deal with general noisy TMUX system, we consider a worst-case reconstruction error, such performance can be very effectively described using H∞ related criteria. To optimize the average (H2) reconstruction performmince while ensuring a certain level of the worst-case error energy over all possible inputs and channel noises, the mixed H2/H∞ optimization is to be sought. If the error system (23) is stable, its H∞ norm is defined as

$$\|\mathcal{E}\|_\infty = \sup_{\|\tilde{s}\|_2 \neq 0} \frac{\|\tilde{e}\|_2}{\|\tilde{s}\|_2}.$$

Moreover, its value is bounded by a prescribed scalar γ if and only if the following inequality holds:

$$\begin{bmatrix} -P & \mathcal{A}^T P & 0 & \mathcal{C}^T \\ P\mathcal{A} & -P & P\mathcal{B} & 0 \\ 0 & \mathcal{B}^T P & -\gamma I & \mathcal{D}^T \\ \mathcal{C} & 0 & \mathcal{D} & -\gamma I \end{bmatrix} < 0.$$

Proof. Equation (31) can be easily derived by applying the Schur complements and the well-known bounded real lemma. Then the mixed H2/H∞ optimization can be solved as follows. Theorem 2. Give a scalar γ > 0, the mixed H2 and H∞ reconstruction problem is solvable if and only if the H∞ reconstruction problem is solvable. In this situation, the optimal mixed H2 and H∞ receivers can be obtained by the following convex optimization:

$$\|\mathcal{E}\|_2^2 = \min_{S,Q,P,\mathcal{C}^h,\mathcal{D}^h} \operatorname{trace}(S)$$

subject to LMIs (29), and (31), with $S = S^T$, $Q = Q^T$, and $P = P^T$.

In [17], a mixed H2/H method is recommended for the design of IIR receivers for a noisy TMUX system; this is an important point to keep in mind. Since the same Lyapunov matrix is used for both the H2 and H performances, the technique in [17] is often conservative. In other words, the best that can be done is a suboptimal blend of H2 and H reception (an upper limit on H2 performance). For TMUX systems, we have presented a hybrid H2/H design using convex optimization, which allows for distinct Lyapunov matrices Q and P for the H2 and H performances, respectively. In other words, Theorem 2's conclusion is both essential and sufficient. In other words, it will result in the best possible answer rather than a less desirable one.

Crosstalk attenuation optimization using H

The crosstalk issue will be addressed here using a H optimization strategy. There are two main motivations for researching crosstalk attenuation using the H method. To begin, crosstalk is a common issue in TMUX systems; for instance, it is the main bottleneck for DSL services because it occurs when numerous services use the same telephone wire to send their signals. For example, the British telecommunication specifications stipulate that a 60-channel TMUX must have at least 60 dB of interchannel crosstalk attenuation [8], a less stringent requirement than crosstalk cancellationla tion that results in lower implementation costs. The second is that complete crosstalk cancellation is difficult to achieve in a TMUX system due to modeling uncertainty [12]. This uncertainty stems from a number of factors. With H optimization, crosstalk may be minimized even under the most extreme conditions.

Definition of the Issues

Crosstalk, as previously mentioned, is the influence of inputs from other bands on the outputs from the ith band, with i=0, 1,..., M1. Using the polyphase identity, we can write Pij(z) = Hi(z)C(z)Fj(z) and denote Gij(z) as the 0th polyphase component of Pij(z) for the TMUX system seen in Figure 2. Then, the ith band's output is represented as

$$\hat{S}_i(z) = G_{ii}(z)S_i(z) + \sum_{j=0, j\neq i}^{M-1} G_{ij}(z)S_j(z) = \hat{S}_{ii}(z) + \hat{S}_{c,i}(z),$$

here Si(z) is the z-transform of si(k) and S c,i(z) is due to the inputs of other bands and is termed as crosstalk in the ith band. In general, the crosstalk in the ith band is composed of (M − 1) leakages from (M − 1) input sj, j = 0, ... , i − 1, i + 1, ... , M − 1. However, this can be simplified considerably if we assume that crosstalk only appears between adjacent channels [3], that is, for a TMUX system, Hi(z) and Fi(z) have the same frequency support domain and Hi(z)Hj(z) ≈ 0 for |i − j| > 1 (nonadjacent filters practically do not overlap). This means that the expression of the ith band crosstalk distortion sc,i(n) for 1 ≤ i ≤ M − 2 contains two significant terms as Fi practically overlaps only with Fi−1 and Fi+1. For i = 0 or i = M − 1 it contains only one significant term as F0 overlaps only with F1 and FM−1 with FM−2. We will now derive a state-space representation for each crosstalk by a lifting approach, it is clear that such representation is a special case of (23), by ignoring the delays and only considering si−1(k), and si+1(k) being sources of the ith crosstalk output. Let Fi denote the mapping (si−1,si+1) → sc,i in the system of Figure 3

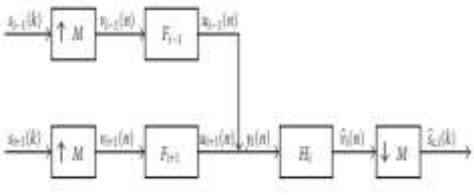

Figure 3: Composition of the ith crosstalk

Denote

$$s_{c,i}(k) = \begin{bmatrix} s_{i-1}(k) \\ s_{i+1}(k) \end{bmatrix}.$$

Following the similar derivation as above, the crosstalk of the ith band is given by

$$(\mathcal{E}_{c,i}) : \begin{aligned} \mathcal{X}_{c,i}(k+1) &= \mathcal{A}_{c,i}\mathcal{X}_{c,i}(k) + \mathcal{B}_{c,i}s_{c,i}(k), \\ \hat{s}_{c,i}(k) &= \mathcal{C}_{c,i}\mathcal{X}_{c,i}(k) + \mathcal{D}_{c,i}s_{c,i}(k), \quad i = 1, \ldots, M-2, \end{aligned}$$

Where is the state victor

$$\mathcal{X}_{c,i}(k) = \left[\overline{x}_{i-1}^{fT}(k) \;\; \overline{x}_{i+1}^{fT}(k) \;\; \overline{x}_{i}^{hT}(k)\right]^T,$$

$$\begin{aligned} \mathcal{A}_{c,i} &= \begin{bmatrix} \overline{A}_{f,i-1} & 0 & 0 \\ 0 & \overline{A}_{f,i+1} & 0 \\ \overline{B}_{h,i}\overline{C}_{f,i-1} & \overline{B}_{h,i}\overline{C}_{f,i+1} & \overline{A}_{h,i} \end{bmatrix}, \\ \mathcal{B}_{c,i} &= \begin{bmatrix} \overline{B}_{f,i-1} & 0 \\ 0 & \overline{B}_{f,i+1} \\ \overline{B}_{h,i}\overline{D}_{f,i-1} & \overline{B}_{h,i}\overline{D}_{f,i+1} \end{bmatrix}, \qquad (36) \\ \mathcal{C}_{c,i} &= \left[D_{h,i}C_{f,i-1} \;\; D_{h,i}C_{f,i+1} \;\; C_{h,i}\right], \\ \mathcal{D}_{c,i} &= D_{h,i}\left[D_{f,i-1} \;\; D_{f,i+1}\right] \end{aligned}$$

with $\mathcal{A}_{c,i} \in \mathcal{R}^{(2l_f+l_h)\times(2l_f+l_h)}$, $\mathcal{B}_{c,i} \in \mathcal{R}^{(2l_f+l_h)\times 2}$, $\mathcal{C}_{c,i} \in \mathcal{R}^{1\times(2l_f+l_h)}$, and $\mathcal{D}_{c,i} \in \mathcal{R}^{1\times 2}$.

The state-space realizations for the crosstalks in 0th and $(M-1)$th bands are

$$\begin{aligned} (\mathcal{E}_{c,0}) : \mathcal{X}_0(k+1) &= \mathcal{A}_{c,0}\mathcal{X}_0(k) + \mathcal{B}_{c,0}s_1(k), \\ \hat{s}_{c,0}(k) &= \mathcal{C}_{c,0}\mathcal{X}_0(k) + \mathcal{D}_{c,0}s_1(k); \\ (\mathcal{E}_{c,M-1}) : \mathcal{X}_{M-1}(k+1) &= \mathcal{A}_{c,M-1}\mathcal{X}_{M-1}(k) + \mathcal{B}_{c,M-1}s_{M-2}(k), \\ \hat{s}_{c,M-1}(k) &= \mathcal{C}_{c,M-1}\mathcal{X}_{M-1}(k) + \mathcal{D}_{c,M-1}s_{M-2}(k), \end{aligned}$$

where the state vector is $\mathcal{X}_l(k) = \left[\overline{x}_l^{fT}(k) \;\; \overline{x}_l^{hT}(k)\right]^T$, $l = 0$, or $M-1$ and

$$\begin{aligned} \mathcal{A}_{c,l} &= \begin{bmatrix} \overline{A}_{f,l} & 0 \\ \overline{B}_{h,l}\overline{C}_{f,l} & \overline{A}_{h,l} \end{bmatrix} \in \mathcal{R}^{(l_f+l_h)\times(l_f+l_h)}, \\ \mathcal{B}_{c,l} &= \begin{bmatrix} \overline{B}_{f,l} \\ \overline{B}_{h,l}\overline{D}_{f,l} \end{bmatrix} \in \mathcal{R}^{(l_f+l_h)\times 1}, \\ \mathcal{C}_{c,l} &= \left[D_{h,l}C_{f,l} \;\; C_{h,l}\right] \in \mathcal{R}^{1\times(l_f+l_h)}, \qquad \mathcal{D}_{c,l} = D_{h,l}D_{f,l} \in \mathcal{R}. \end{aligned}$$

EXAMPLES

Example

1 The challenge of TMUX rebuilding is now being tackled. We take into account the model described in [17], and suggest a hybrid H2/H strategy for designing the receivers. Channel noise (Signal-to-Noise Ratio, SNRc) and reconstruction performance (Signal-to-Noise Ratio, SNRr i) are defined first.

$$\mathrm{SNR}^c = 10\log_{10}\frac{\sum_{k=0}^{\infty} p^2(k)}{\sum_{k=0}^{\infty} r^2(k)},$$

$$\mathrm{SNR}_i^r = 10\log_{10}\frac{\sum_{k=0}^{\infty} s_i^2(k)}{\sum_{k=0}^{\infty}\left(\hat{s}_i(k) - s_i^d(k)\right)^2}.$$

Then, the outcomes are shown in Table 1 (for the first frequency band). As can be shown, by using distinct Lya punov matrices for the H2 and H performances, our suggested strategy achieves somewhat better reconstruction performances than the conservative method provided in [17]. The more of a limit is placed on how well H performs, the more noticeable the result.

Example .2

Here, we take a look at how well a TMUX system suppresses crosstalk. In [22], a model of a filter bank with three channels is considered, and a filter bank capable of flawless reconstruction is developed. This is why we use a dual system approach for a three-band PR TMUX. With the SNRc set at 20 dB and 10 dB, respectively, and the channel noise having a variance of 2 r = 0.09 and 2 r = 0.9, we optimize the H2 design (Theorem 1) with a H crosstalk restriction to build the receivers.Third theorem. Different restrictions SCRs are described in (39), and the original perfect reconstruction (PR) TMUX system is compared in Table 2. First, the proposed optimal design with a H crosstalk constraint outperforms the PR design in both crosstalk attenuation and reconstruction performance; second, our proposed H constraint can obtain any desired crosstalk attenuation requirement; and third, when a stringent crosstalk attenuation is required, the reconstruction performance could be very poor. It's important to note that the substantial frequency overlapping of the three transmit ters is mostly to blame for the poor overall reconstruction performance in this scenario.

CONCLUSION

In this study, we explore the best way to design receivers for noisy transmultiplexer systems in order to minimize reconstruction errors and keep crosstalk to a minimum. The former is maximized using the H2 method, whereas the latter is formulated and solved using the H method. The simulation results showed that the suggested design outperformed the biorthogonal transmultiplexer design in terms of reconstruction and crosstalk reduction in noisy environments.

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